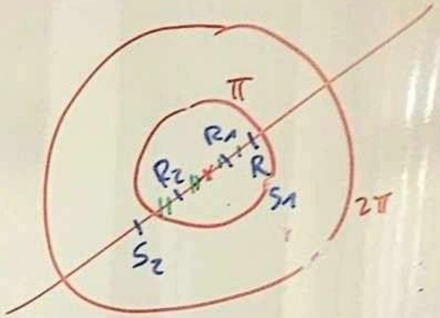
1- Les spineurs sont les racines carrées des rotations (Samuel Bernardet – Dotwave Labs- Chambéry) :

La fonction d'onde d'un objet quantique à 2 degrés de liberté est un spineur. Les spineurs s'enroulent autour du groupe des rotations de l'espace comme un ruban de #Möbius à 3 dimensions, et on peut expérimenter directement cet étrange phénomène topologique en jouant avec la "boule spinorielle"™. On explique souvent que les spineurs sont les racines carrées des vecteurs ou de la géométrie, avec des justifications longues comme un discours de Fidel Castro. Dans le cas d'école des spineurs du groupe des rotations de l'espace, un seul tableau blanc suffit à montrer que : les spineurs sont, à une substitution de générateur près, les racines carrées des rotations de l'espace.

Rotation générateurs $\vec{g}$ $\left| \begin{array}{l} g_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \\ g_y = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \\ g_z = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \right.$ $R = e^{\vec{g}\cdot\vec{n}\,\theta}$ $\vec{n} \left| \begin{array}{l} n_x \\ n_y \\ n_z \end{array} \right.$ $|\vec{n}| = 1$



Spineur $\vec{q}$ $\left| \begin{array}{l} i = \begin{bmatrix} 0 & -h \\ -h & 0 \end{bmatrix} \\ j = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \\ k = \begin{bmatrix} -h & 0 \\ 0 & h \end{bmatrix} \end{array} \right.$ $(h^2 = -1)$ $S = e^{\vec{q}\cdot\vec{n}\,\frac{\theta}{2}}$

$$R = R(\vec{n},\theta) = e^{\vec{g}\cdot\vec{n}\,\theta} \nearrow S_1 = e^{\vec{q}\cdot\vec{n}\,\frac{\theta}{2}} \longrightarrow R_1 = e^{\vec{g}\cdot\vec{n}\,\frac{\theta}{2}}$$

$$\searrow S_2 = e^{\vec{q}\cdot\vec{n}\left(\frac{\theta}{2}-\pi\right)} \longrightarrow R_2 = e^{\vec{g}\cdot\vec{n}\left(\frac{\theta}{2}-\pi\right)}$$

$S_2 = -S_1$

Remplacement de $\vec{q}$ par $\vec{g}$

↳ $R_1^2 = R_2^2 = R$ et $R_1 \neq R_2$

$R_1 =$ "$\sqrt{R}$" $R_2 =$ "$-\sqrt{R}$"

Les 2 spineurs associés à une rotation donnent directement les 2 racines de R en substituant $\vec{q}$ par $\vec{g}$

Le cas singulier $\theta = 0$ a une indétermination sur $\vec{n}$ ce qui correspond à une infinité de

Spinors can be viewed as the "square roots" of vectors (although this is inaccurate and may be misleading; they are better viewed as "square roots" of sections of vector bundles – in the case of the exterior algebra bundle of the cotangent bundle, they thus become "square roots" of differential forms).

Wikipedia

No one fully understands spinors. Their algebra is formally understood, but their geometrical significance is mysterious. In some sense they describe the “square root” of geometry and, just as understanding the concept of square root of –1 took centuries, the same might be true of spinors.

— *Sir Michael Atiyah, British mathematician*

1. Spineurs sont racines des rotations
    - Spineur
    - Rotations
    - Couverture Double - boule spinorielle
    - Racines de rotation
    - Rotation 2PI = *échange* des racines

2. Spin-statistics
    - Tour de la ceinture de Dirac
    - Tour de la ceinture de Feynman
    - Echange <-> Rot 360 d'une des particules

1. Déformation de l'espace a la spin ½

    Comment faire tourner une sphère des un pot de gelée sans que cela se voit de l'extérieur

① Spineurs

Rotations

Couverture double
→ boule

Racine carrée

360° ⟺ échange des racines

② Spin Statistics

$$\Psi(r_1, r_2) = \pm \Psi(r_2, r_1)$$

Ceinture de Feynman
(Rappel Dirac
→ Echange ⟺ 360°

③ Deformation espace & Twister
Vid

Rot. Sphère dans bocal
de gelée

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle \qquad a, b \in \mathbb{C} \quad |a|^2+|b|^2 = 1$$

<u>Spineur</u> $\begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{C}^2$

unitaire $|a|^2+|b|^2 = 1$

$$\begin{pmatrix} a \\ b \end{pmatrix} \longleftrightarrow \begin{pmatrix} a & -b^* \\ b & a^* \end{pmatrix} \in SU(2)$$

spineur

SU(2) groupe & variété de dim 3

$s \in SU(2)$ s'écrit $s = e^g$

g générateur $\begin{cases} \text{anti-H} \\ \text{trace nulle} \end{cases}$

ev dim 3

$$g = N_1 i + N_2 j + N_3 k$$

$$i = \begin{pmatrix} 0 & -h \\ -h & 0 \end{pmatrix} \quad j = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad k = \begin{pmatrix} -h & 0 \\ 0 & h \end{pmatrix}$$

$(h^2 = -1)$

quaternions

$$i^2 = j^2 = k^2 = ijk = -1$$

$$s = e^{\vec{q}\cdot\vec{n}\,\alpha} \qquad \vec{N}\begin{vmatrix} N_1 \\ N_2 \\ N_3 \end{vmatrix} = \alpha \begin{vmatrix} n_1 \\ n_2 \\ n_3 \end{vmatrix} = \alpha\vec{n}$$

$$|\vec{n}| = 1$$

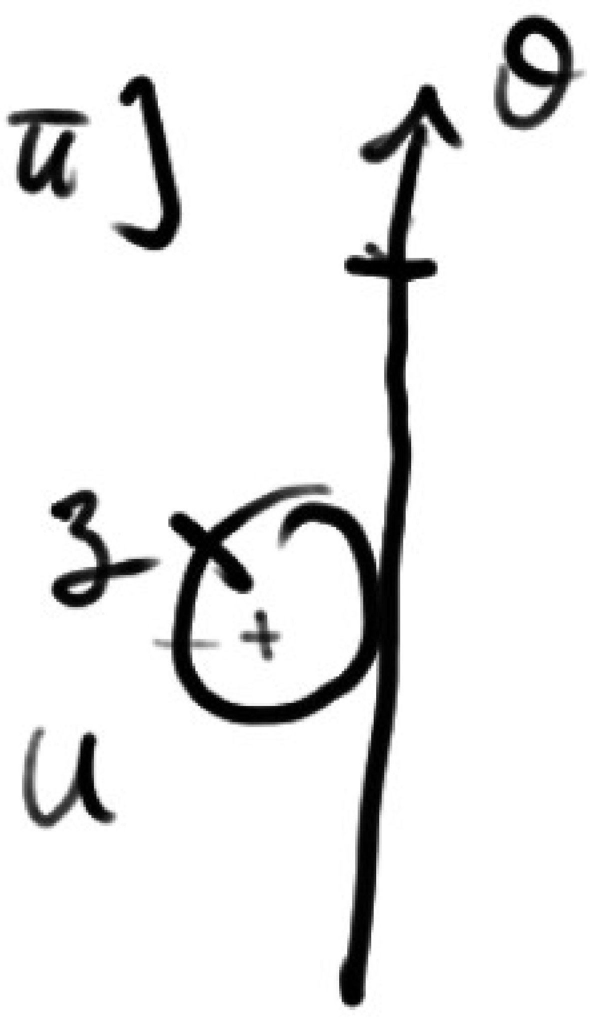
$$(\vec{q}\cdot\vec{n})^2 = -1$$

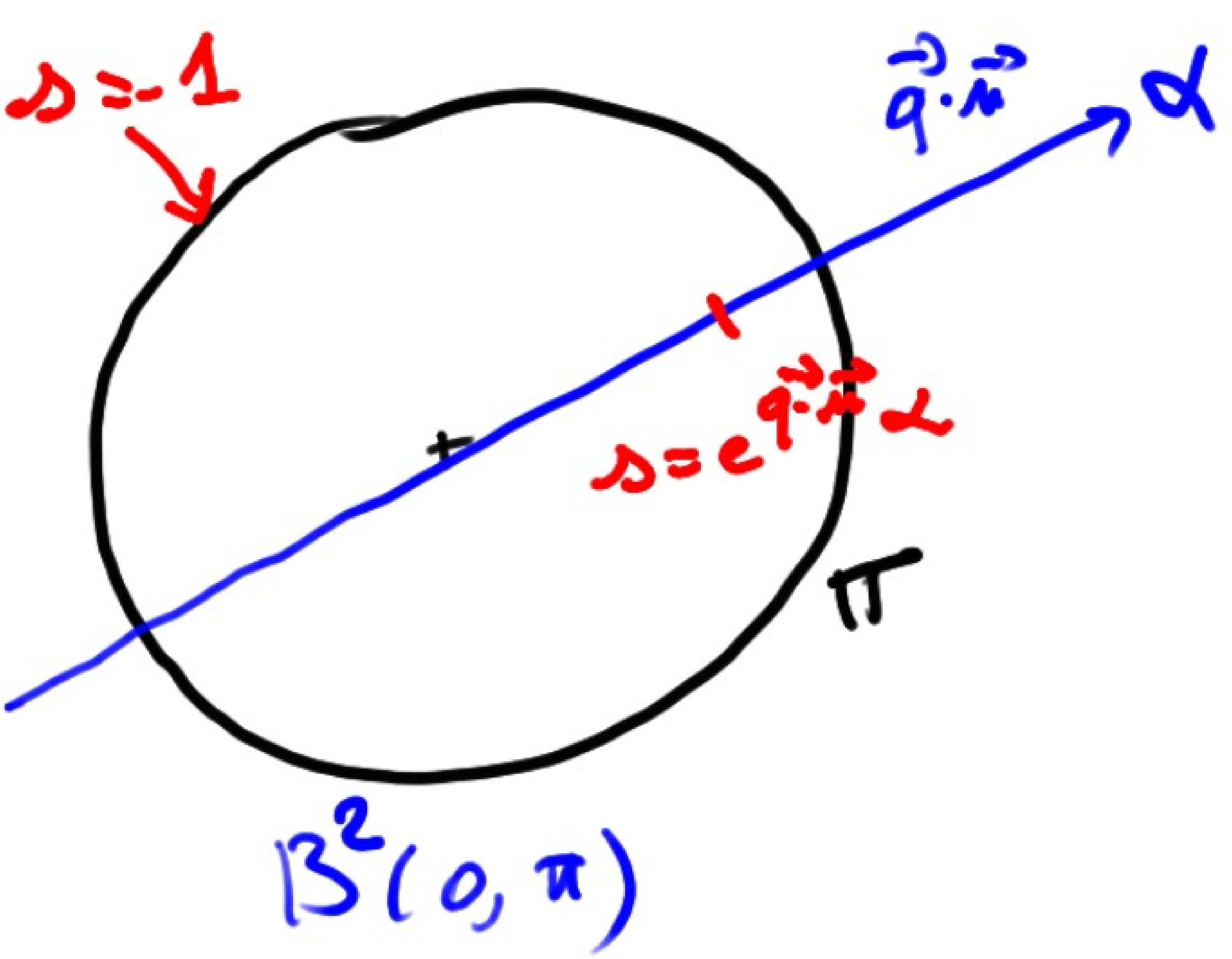
$$e^{\vec{q}\cdot\vec{n}\,\alpha} = \cos\alpha + \vec{q}\cdot\vec{n}\,\sin\alpha$$

$$\left(e^{i\alpha} = \cos\alpha + i\sin\alpha\right)$$

$$\rightarrow \begin{cases} e^{\vec{q}\cdot\vec{n}\,\pi} = -1 & (e^{i\pi} = -1) \\ e^{\vec{q}\cdot\vec{n}\,\frac{\pi}{2}} = \vec{q}\cdot\vec{n} & (e^{i\frac{\pi}{2}} = i) \\ e^{\vec{q}\cdot\vec{n}\,2\pi} = \underline{1} & \end{cases}$$

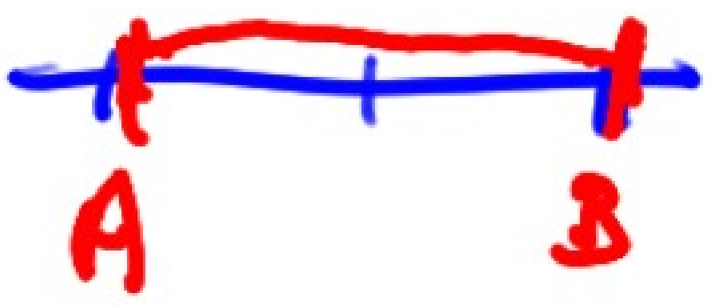
$$s = e^{\vec{q}\cdot\vec{n}\,\alpha} \qquad \alpha \in [-\pi\ \pi]$$

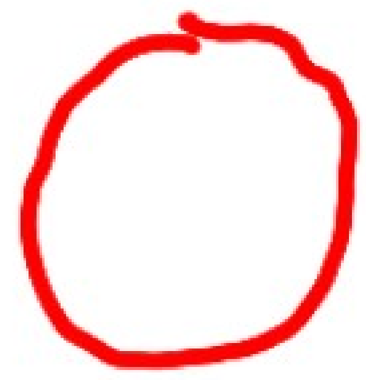




$z \in U \quad z = e^{i\theta}$

$\theta \in [-\pi\ \pi]$



⟺ S^1 

$A = B$

$(B_0, A = B)$

 ⟺ S^2 

$(B_1$, pts de circonf. identifiés$)$

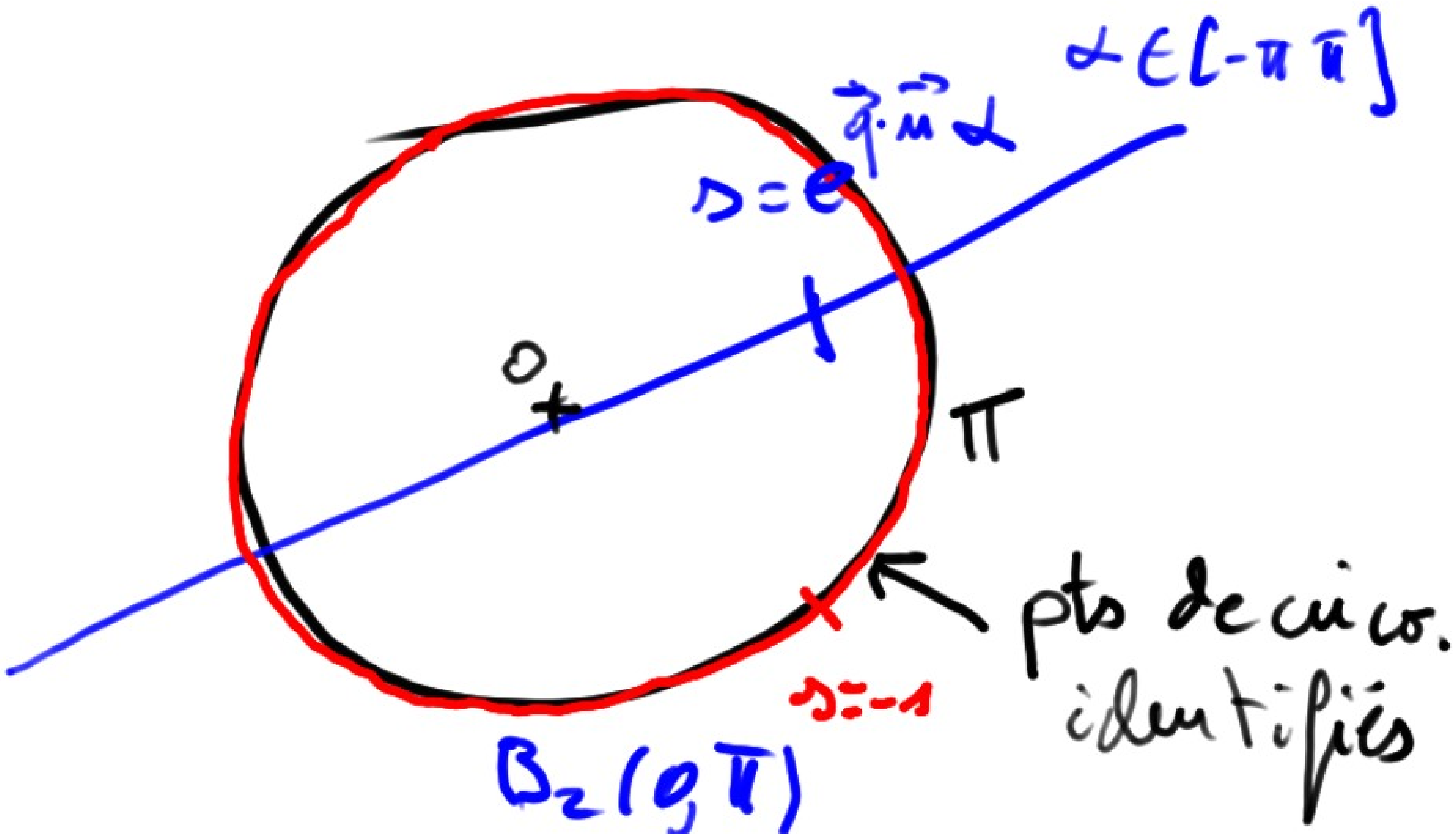
$(B_2$, circo identifiés$)$ ⟺ S^3

Or S^3 est bien isomorphe aux quaternions.

$$q \left| \begin{array}{l} a = x + iy \\ b = w + iz \end{array} \right.$$

$$x^2 + y^2 + w^2 + z^2 = 1$$

$\hookrightarrow q \in S^3$



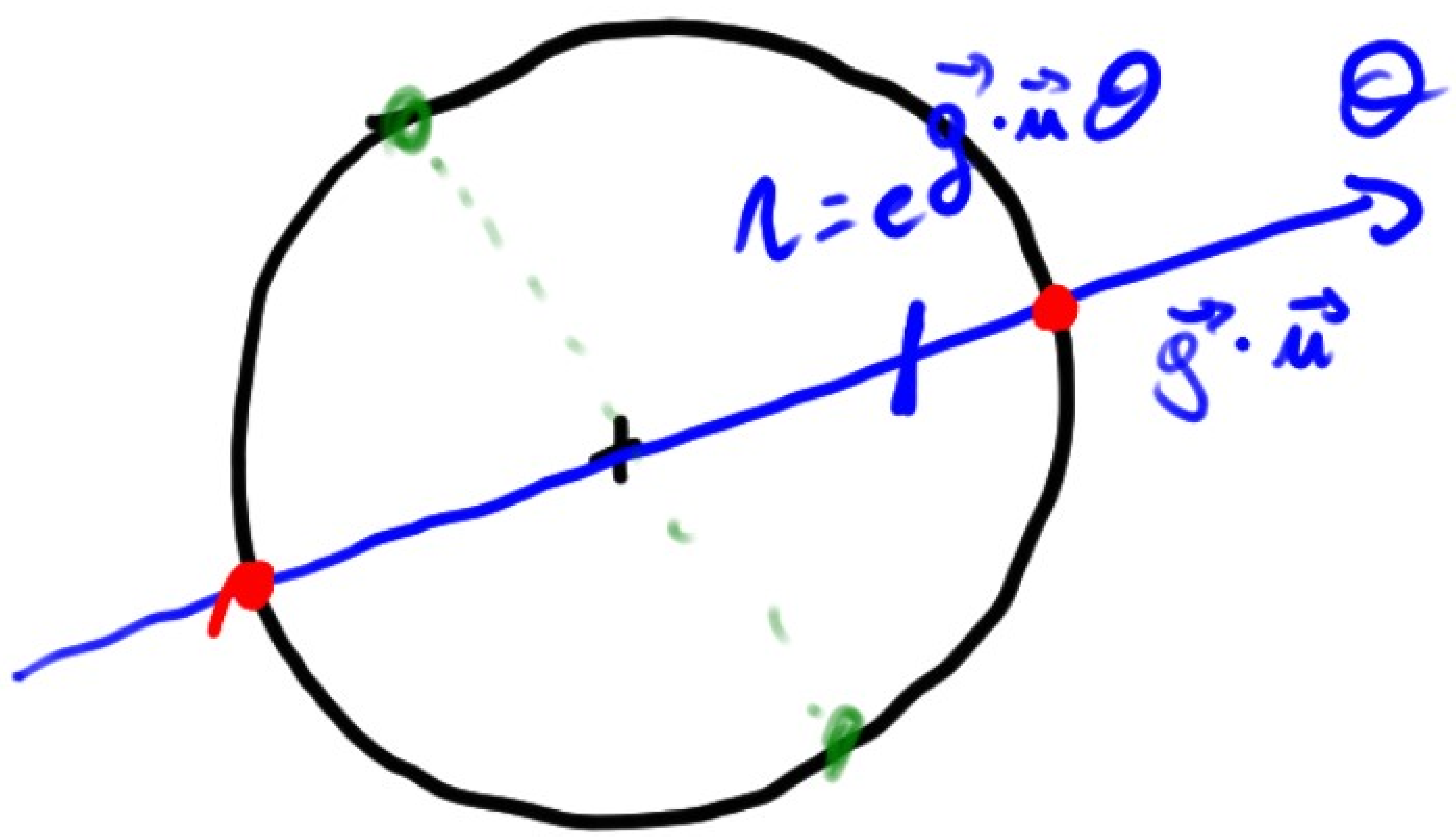
Rotations

$\vec{n}, \theta \quad |\vec{n}| = 1 \quad R(\vec{n}, \theta) \in SO(3)$

$$R(\vec{n}, \theta) = e^{\vec{g}\cdot\vec{n}\,\theta} \qquad \theta \in [-\pi \ \pi]$$

groupe continu
dim 3

$$R(\vec{n}, \pi) = R(-\vec{n}, \pi) \quad .$$



$\vec{g}$ | $g_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

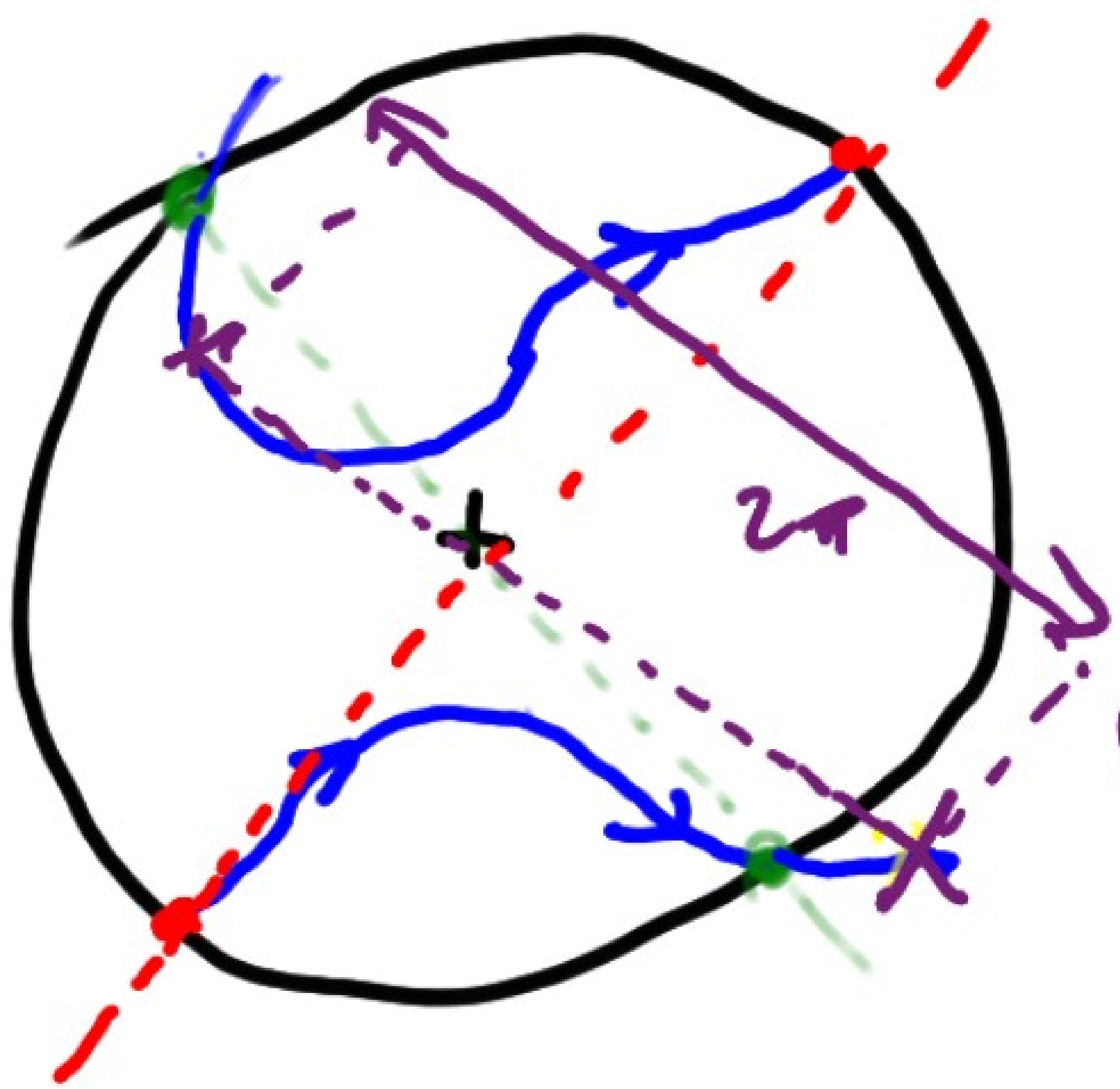
$g_y = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$g_z = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

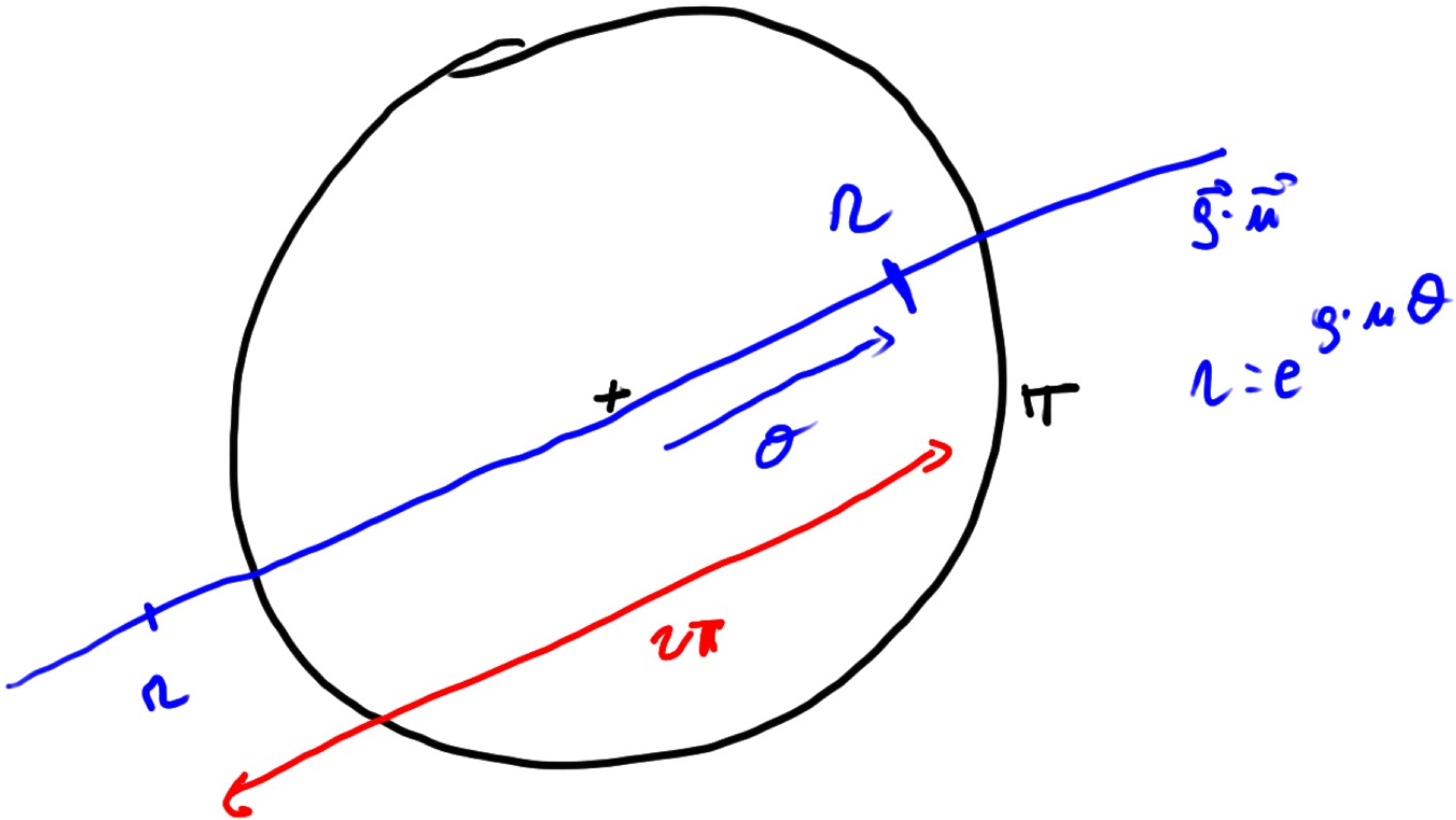
$B^2(0, \pi)$ ac points opposés identifiés

boucle contractibles vs non contractible

$\pi_1 = \mathbb{Z}/2\mathbb{Z}$



$R(\vec{n}, \pi+\alpha) = R(-\vec{n}, \pi-\alpha)$

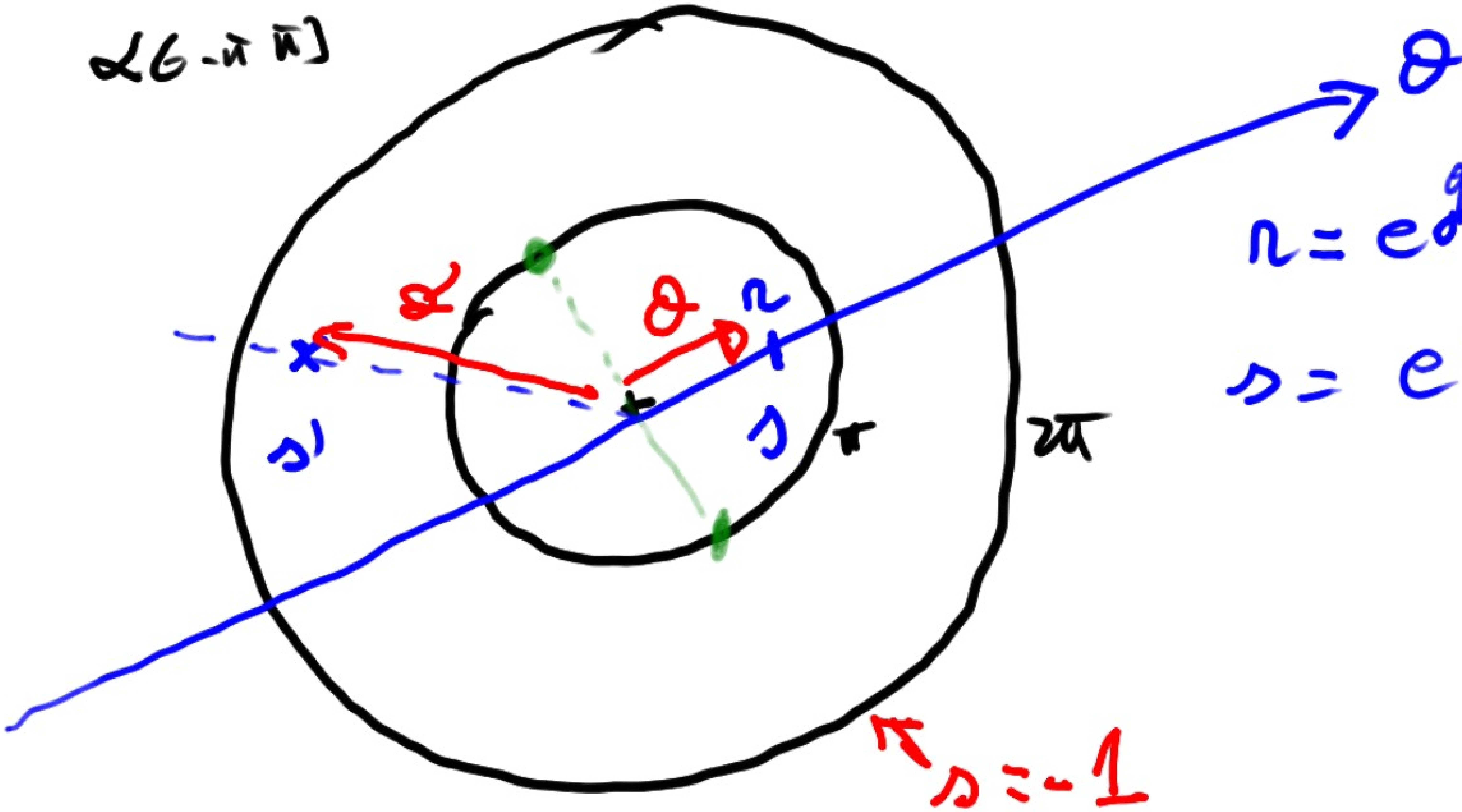


n
θ
π
2π
g·u
n = e^{g·u θ}

Rotation et spineurs sur un schéma

$s = e^{\vec{q}\cdot\vec{n}\alpha}$ $\quad \alpha = \frac{\theta}{2} \quad \theta = 2\alpha \in [-2\pi \ \ 2\pi]$

$\alpha \in [-\pi \ \ \pi]$



$r = e^{q\cdot n\,\theta}$

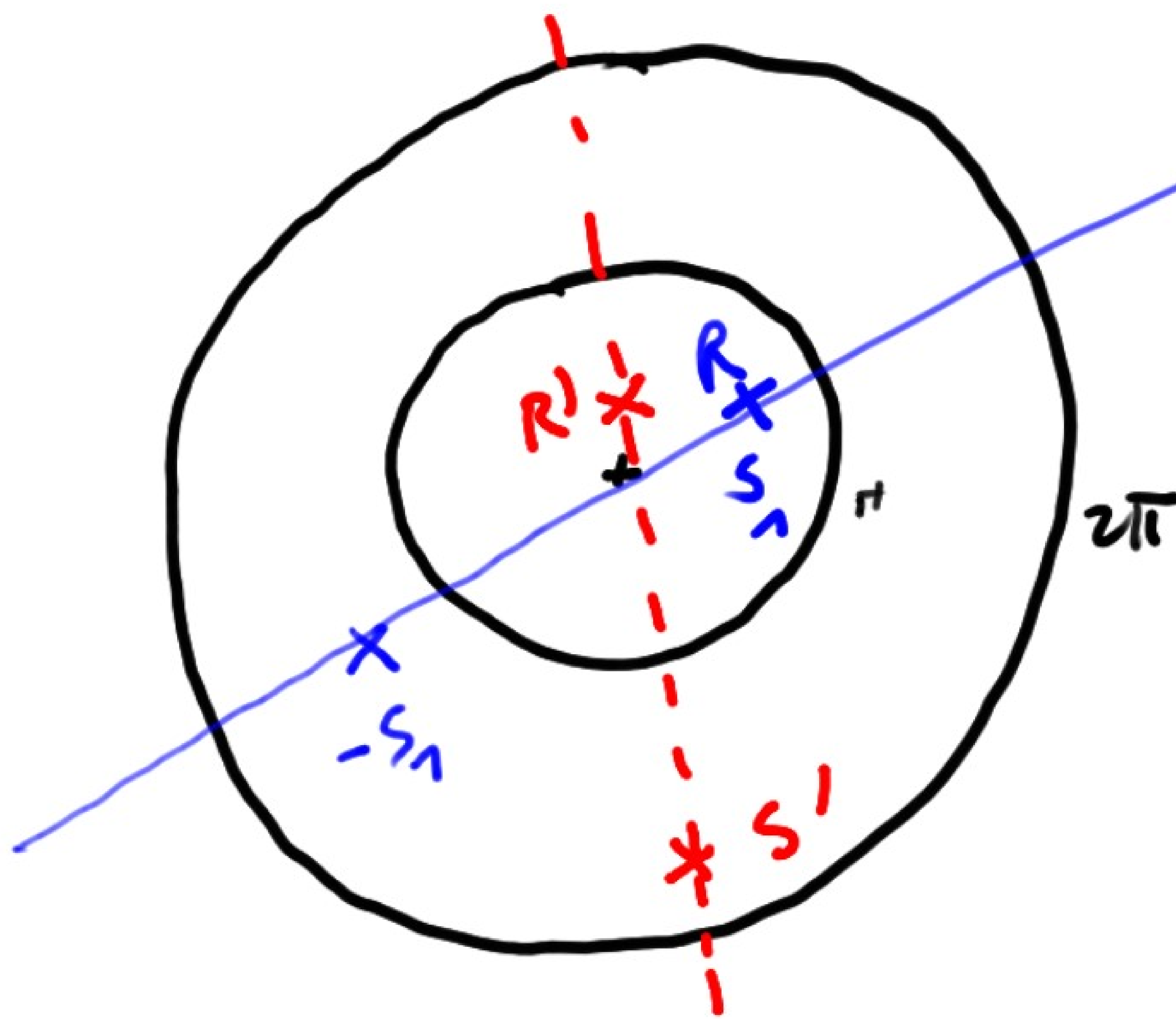
$s = e^{\vec{q}\cdot\vec{n}\,\frac{\theta}{2}}$

$s \in SU(2)$ $\quad \pi : SU(2) \longrightarrow SO(3)$

$e^{\vec{g}\cdot\vec{n}\frac{\theta}{2}} = s \longmapsto \pi(s)$

$\pi(s) = e^{\vec{g}\cdot\vec{n}\,\theta}$ si $\theta \in [0, \pi]$

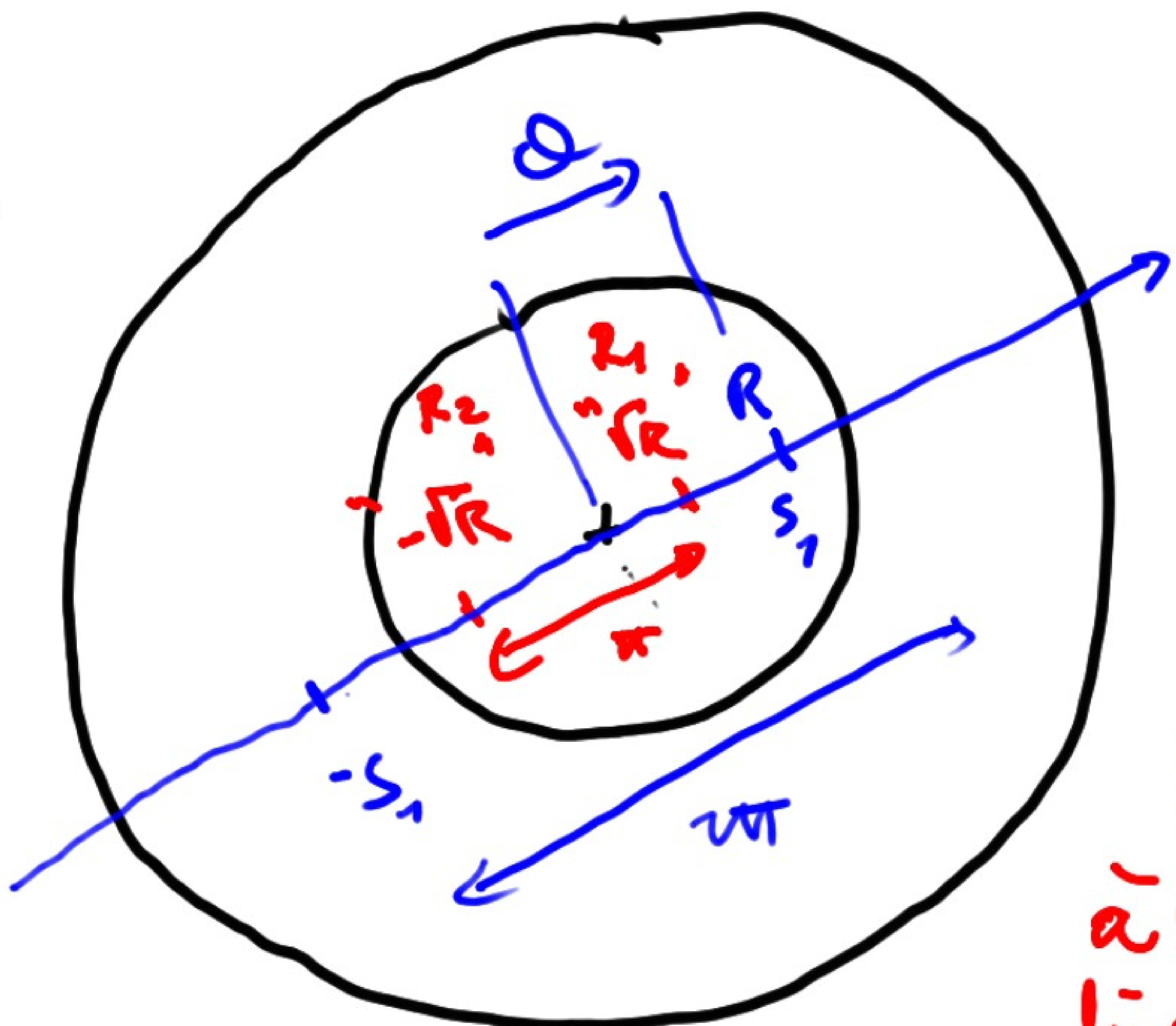
$\pi(s) = e^{\vec{g}\cdot\vec{n}(\theta - 2\pi)}$ si $\theta \in [\pi, 2\pi]$



$\pi^{-1}(R) = \{S_1, S_2 = -S_1\}$ exple avec boule

$$R = R(\vec{u}\,\theta) = e^{\vec{g}\cdot\vec{u}\,\theta} \xrightarrow{\pi^{-1}} s_1 = e^{\vec{q}\cdot\vec{u}\frac{\theta}{2}} \rightarrow R_1 = e^{\vec{g}\cdot\vec{u}\frac{\theta}{2}}$$

$$\searrow s_2 = e^{\vec{q}\cdot\vec{u}\left(\frac{\theta-2\pi}{2}\right)} = -s_1 \rightarrow R_2 = e^{\vec{g}\cdot\vec{u}\left(\frac{\theta-2\pi}{2}\right)} = e^{\vec{g}\cdot\vec{u}\left(\frac{\theta}{2}-\pi\right)}$$



$$R_1^2 = R_2^2 = R \quad \text{et} \quad R_1 \neq R_2$$

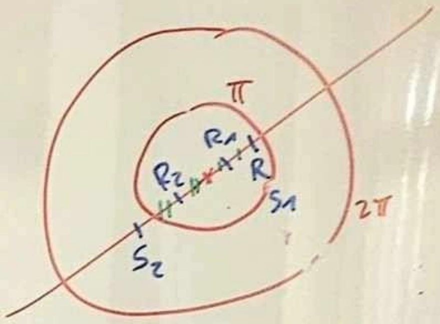
$$R_1 = \text{"}\sqrt{R}\text{"} \qquad R_2 = \text{"}-\sqrt{R}\text{"}$$

Les spineurs associés à une rotation donnent directement les racines de R en remplaçant $\vec{q}$ par $\vec{g}$

cas singulier $\theta = 0$
→ indétermination sur $\vec{u}$
(⇒ R(2π)) infinité de racines

Rotation générateur $\vec{g}$: $g_x = \begin{bmatrix}0&0&0\\0&0&-1\\0&1&0\end{bmatrix}$; $g_y = \begin{bmatrix}0&0&-1\\0&0&0\\1&0&0\end{bmatrix}$; $g_z = \begin{bmatrix}0&1&0\\-1&0&0\\0&0&0\end{bmatrix}$

$$R = e^{\vec{g}\cdot\vec{n}\,\theta} \qquad \vec{n}\,\begin{vmatrix} n_x \\ n_y \\ n_z \end{vmatrix} \quad |\vec{n}| = 1$$



Spineur

$\vec{q}$: $i = \begin{bmatrix}0&-h\\-h&0\end{bmatrix}$; $j = \begin{bmatrix}0&-1\\1&0\end{bmatrix}$; $k = \begin{bmatrix}-h&0\\0&h\end{bmatrix}$ $(h^2 = -1)$

$$S = e^{\vec{q}\cdot\vec{n}\,\frac{\theta}{2}}$$

$$R = R(\vec{n},\theta) = e^{\vec{g}\cdot\vec{n}\,\theta}$$

$\nearrow$ $S_1 = e^{\vec{q}\cdot\vec{n}\,\frac{\theta}{2}} \longrightarrow R_1 = e^{\vec{g}\cdot\vec{n}\,\frac{\theta}{2}}$

$\searrow$ $S_2 = e^{\vec{q}\cdot\vec{n}\,(\frac{\theta}{2}-\pi)} \longrightarrow R_2 = e^{\vec{g}\cdot\vec{n}\,(\frac{\theta}{2}-\pi)}$

$S_2 = -S_1$

Remplacement de $\vec{q}$ par $\vec{g}$

↳ $R_1^2 = R_2^2 = R$ et $R_1 \neq R_2$

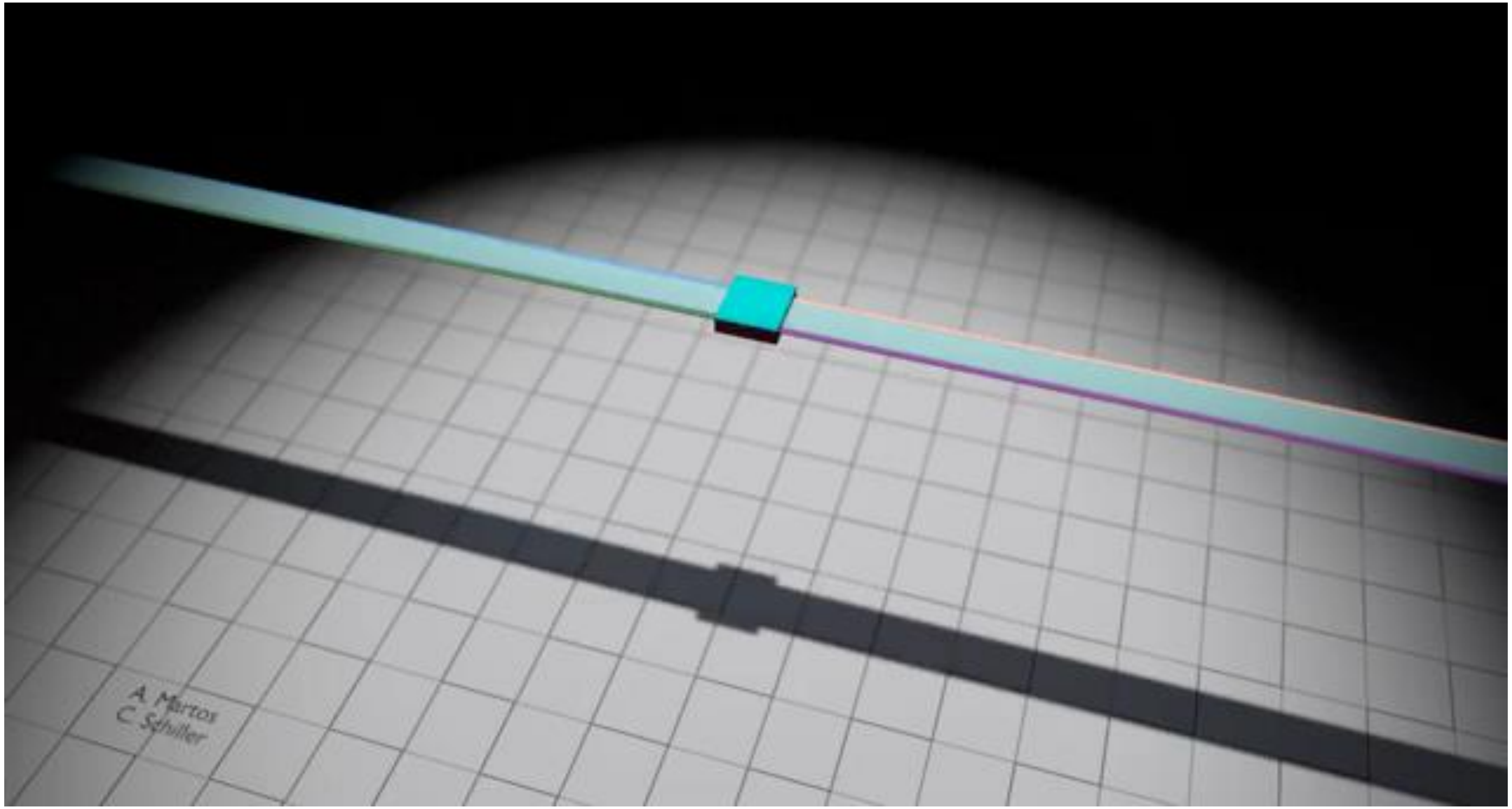
$R_1 =$ "$\sqrt{R}$" $\quad R_2 =$ "$-\sqrt{R}$"

Les 2 spineurs associés à une rotation donnent directement les 2 racines de R en substituant $\vec{q}$ par $\vec{g}$

Le cas singulier $\theta \equiv 0$ a une indétermination sur $\vec{n}$ ce qui correspond à une infinité de ...

2 – Spin Statistique

Rappel : Tour de la ceinture de Dirac



https://vimeo.com/62228139

Théorème Spin Statistique

Fermion <-> Fonction d’échange antiSym
(D’où principe exclusion Pauli)

Boson <-> fonction d’échange Symétrique
(Accumulation de particule dans le même état :
Laser, condensat de bose einstein)

THE REASON FOR ANTIPARTICLES

Richard P. Feynman

The title of this lecture is somewhat incomplete because I really want to talk about two subjects: first, why there are antiparticles, and, second, the connection between spin and statistics. When I was a young man, Dirac was my hero. He made a breakthrough, a new method of doing physics. He had the courage to simply guess at the form of an equation, the equation we now call the Dirac equation, and to try to interpret it afterwards. Maxwell in his day got his equations, but only in an enormous mass of 'gear wheels' and so forth.

I feel very honored to be here. I had to accept the invitation, after all he was my hero all the time, and it is kind of wonderful to find myself giving a lecture in his honor.

1986 Dirac Memorial Lecture

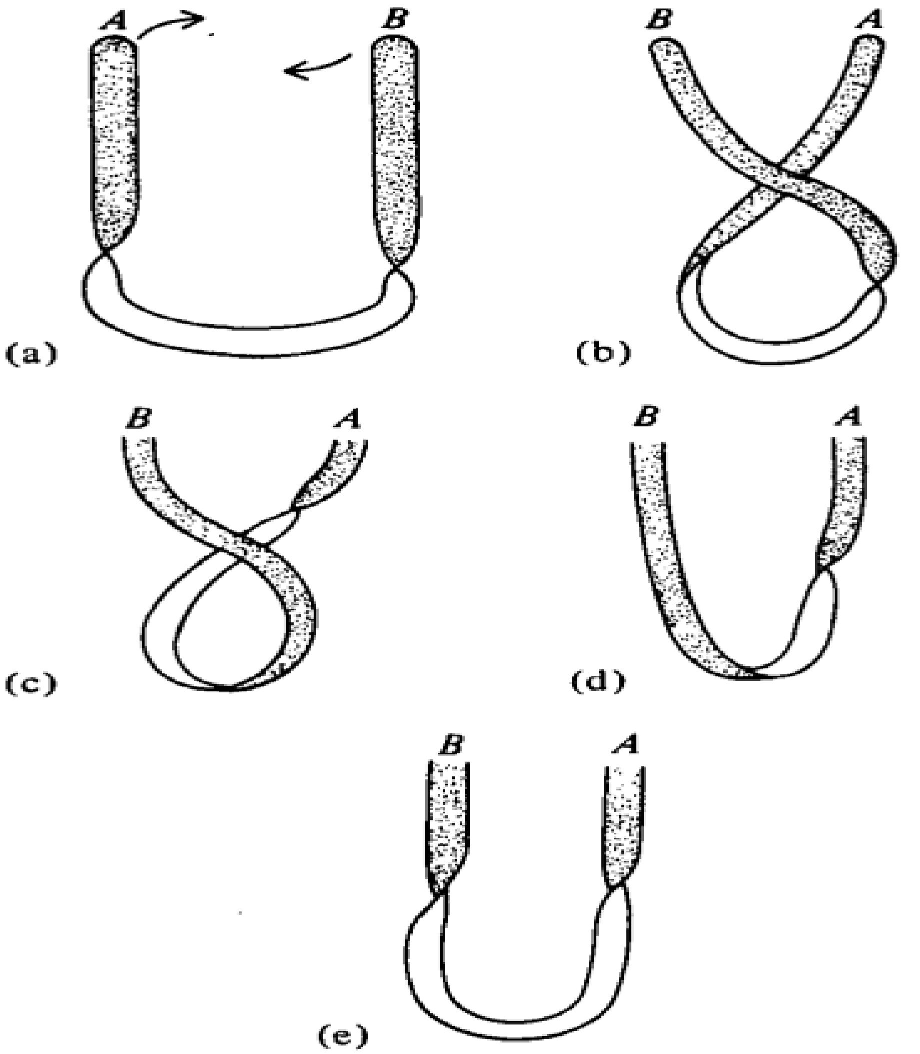


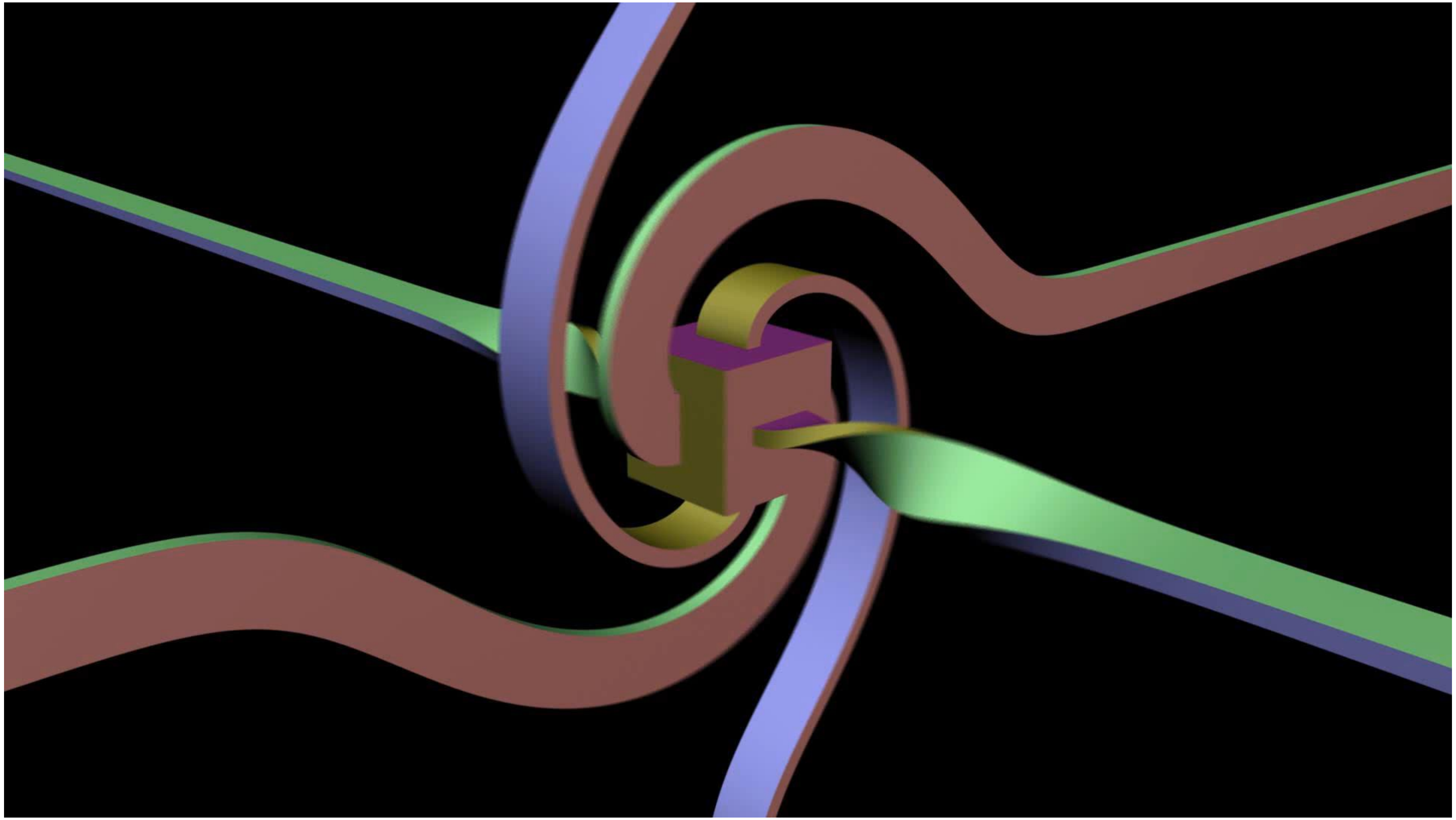
Fig. 16 In the sequence (a) to (e) the ends of the belt have been reversed in position. Note that the twist on the right-hand side in (e) comes out opposite that in (a). To restore it completely, an additional 360° turn of the right belt around the vertical would be necessary.

1986 Dirac Memorial Lecture

Since exchange implies such a 360° rotation of one object relative to the other, there is every reason to expect the (-1) phase factor occasioned by such a rotation for exchange of half integral spin objects.

→Comment Transposer cela en terme de fonction d’onde spatiale avec Spin ?

3 - Déformation de l’espace avec Spin 1/2



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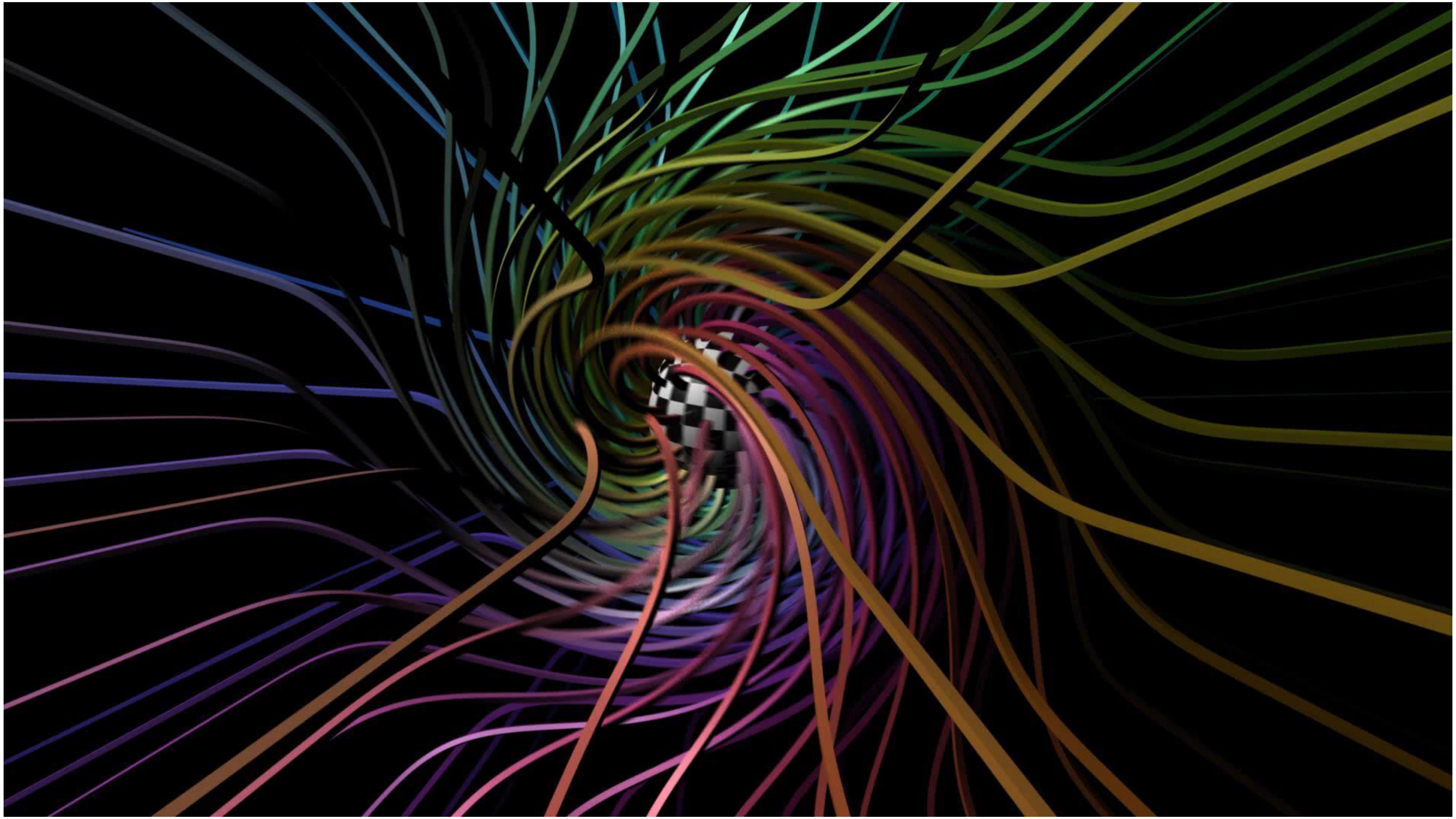

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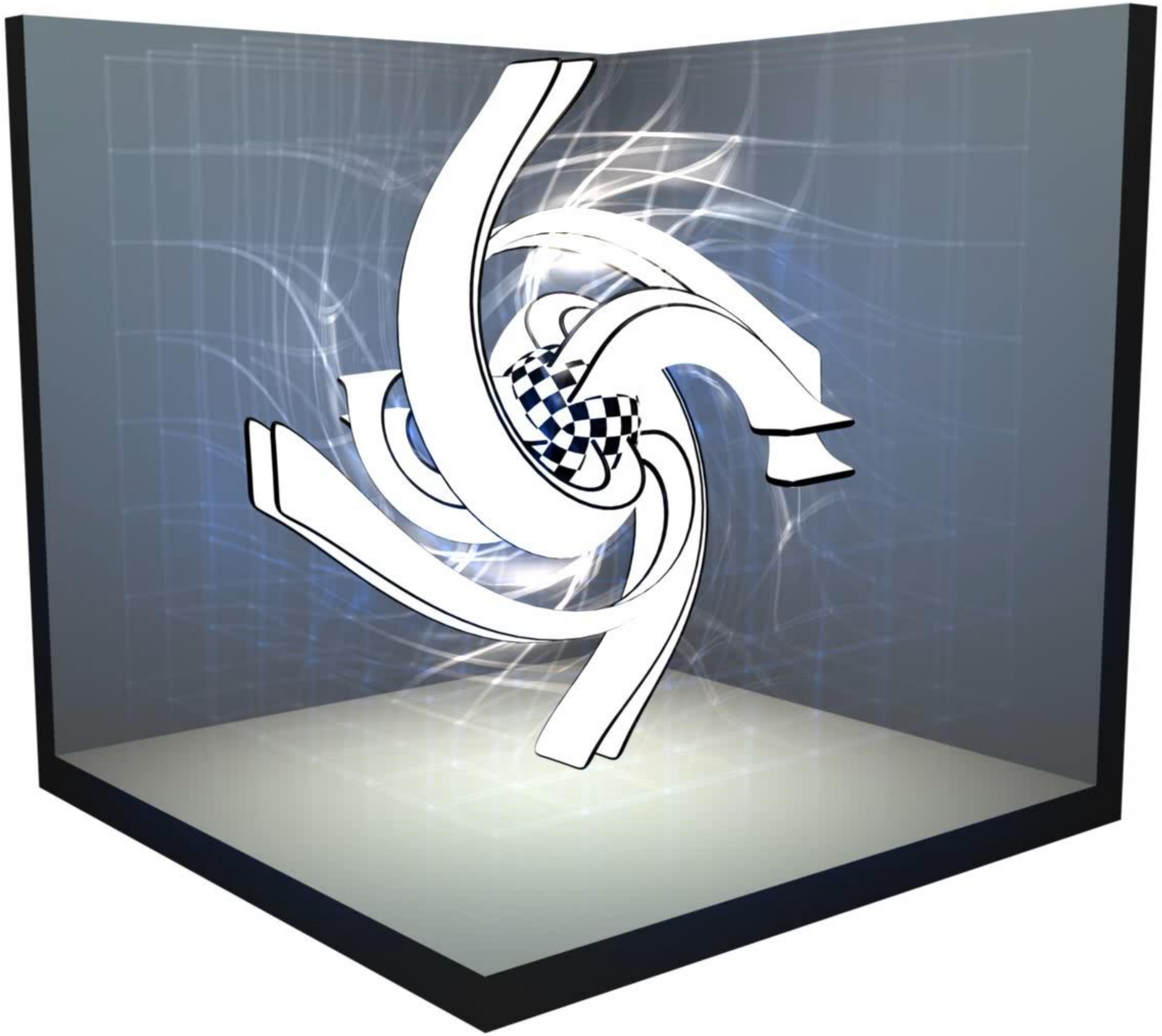

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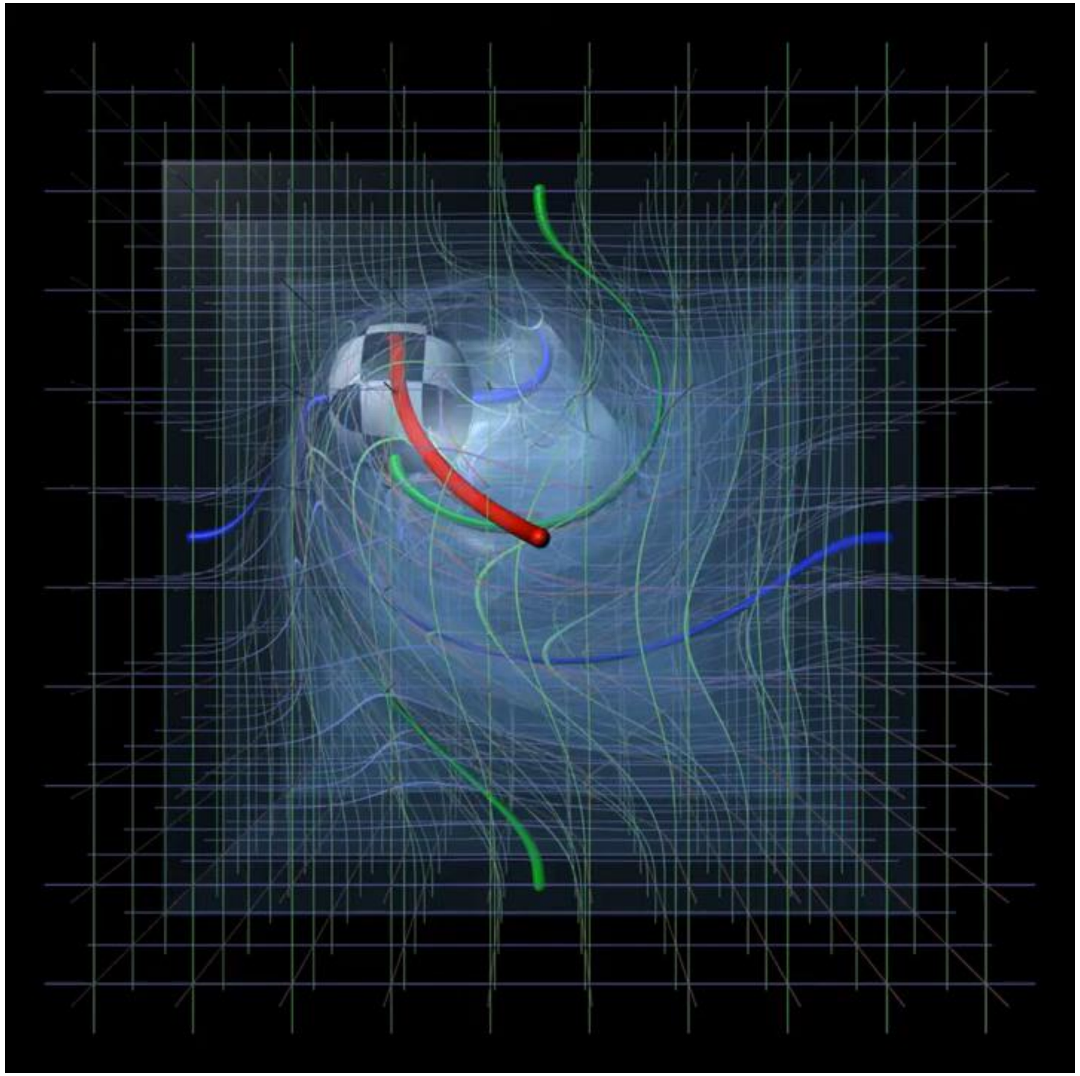

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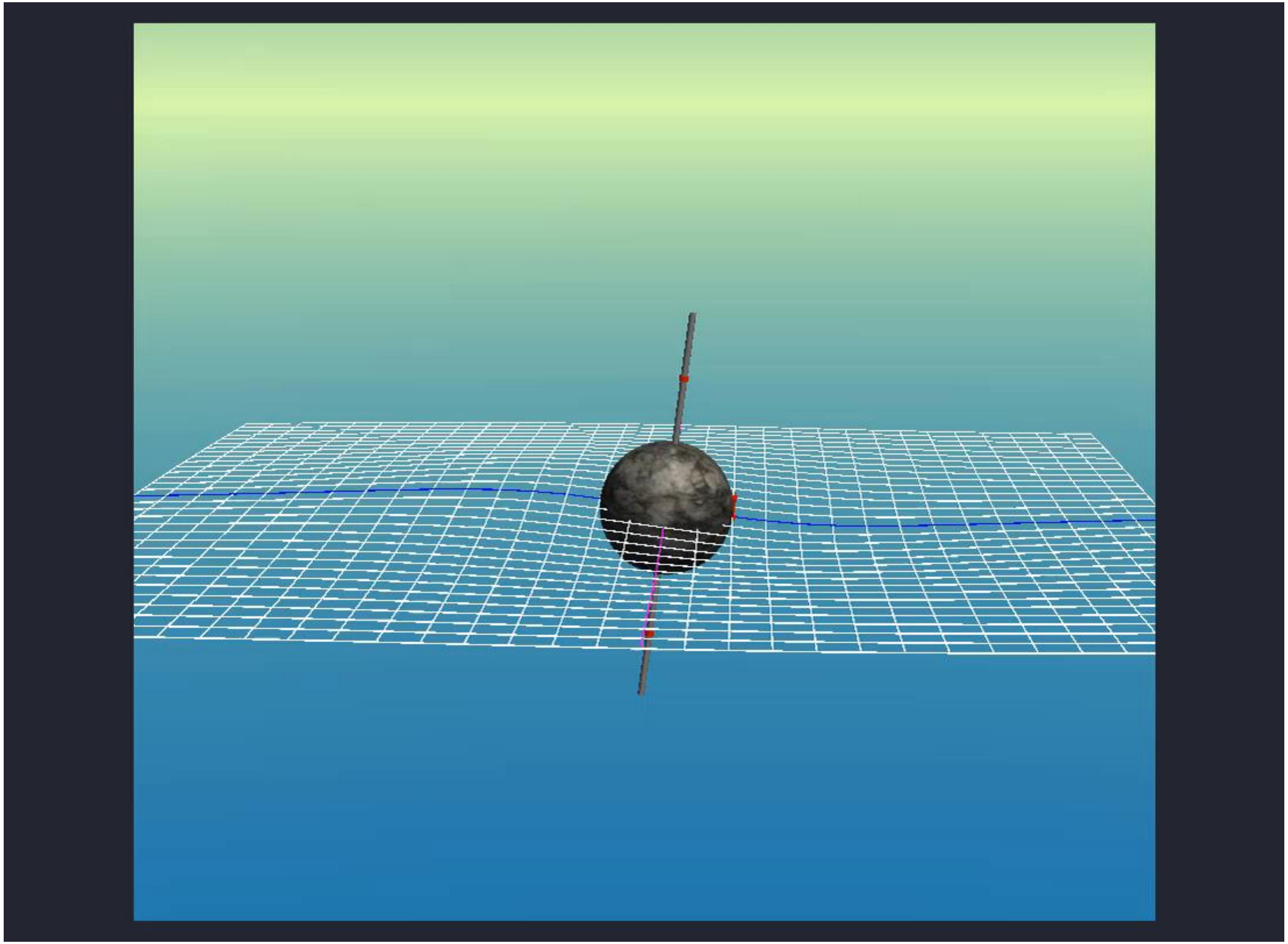
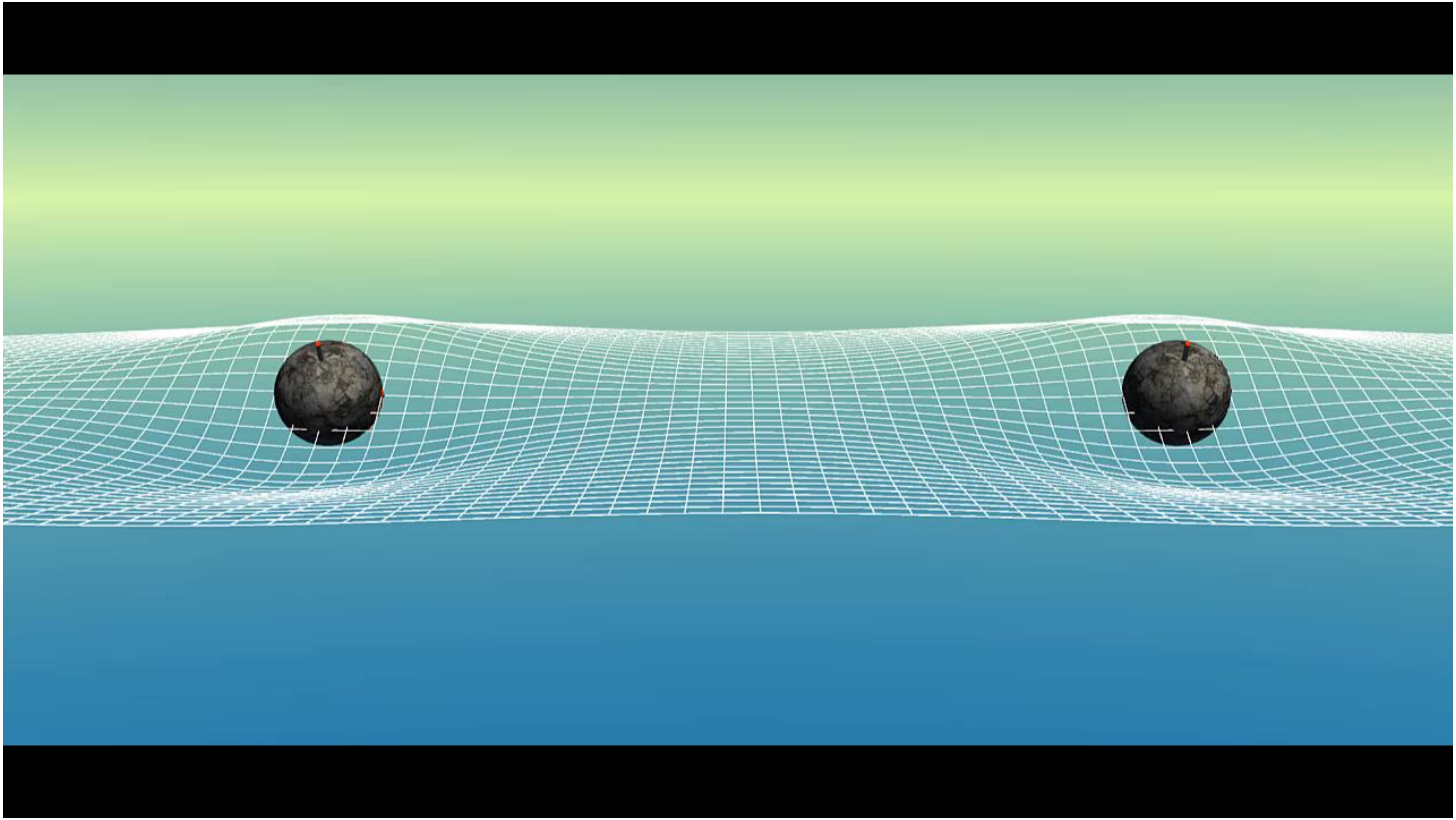


https://elastic-universe.org/wave-and-spin-visualizations/

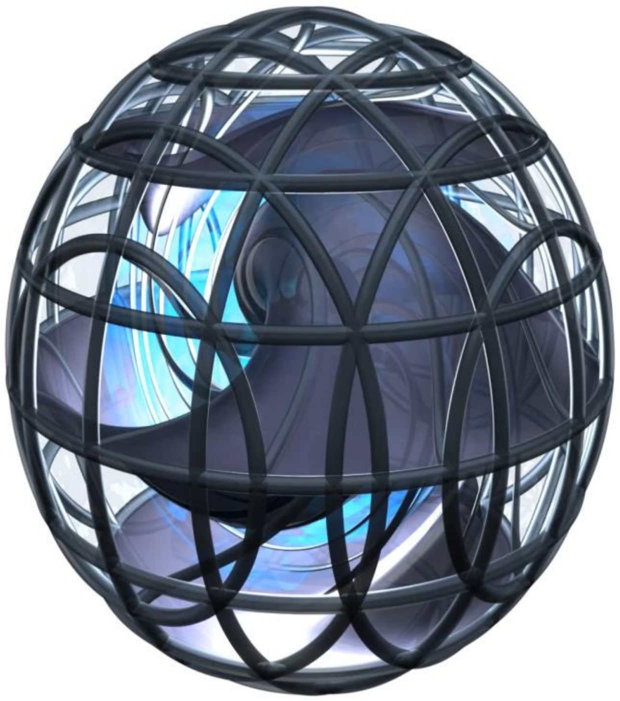


https://elastic-universe.org/wave-and-spin-visualizations/

L'échange de 2 particules est équivalente à la rotation de 360 degré d'une des 2 particules

- ➔Formalisation des l'espace de 2 particules avec spin dans R3

- ➔Visualisation de l'échange par déformation d'un réseau 3D : cela est il équivalent à une rotation 360 ?



Mouchoir de Dirac

https://www.youtube.com/@JasonHise64/videos