Recurrent Neural Networks

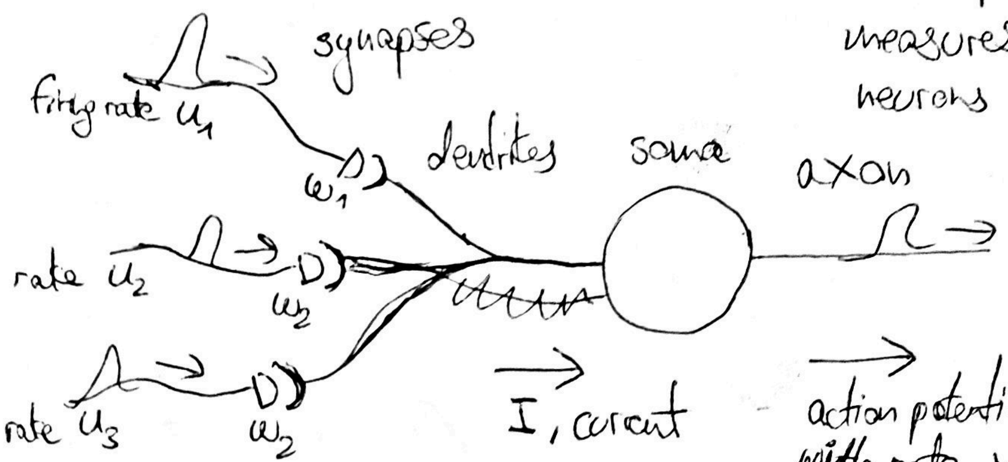
Spiking Models $\longrightarrow$ **Firing rate Models**

Spiking Models:

- non-differentiable, calculational difficulties
- deterministic, but not all inputs are known so a stochastic model would be better

Firing rate Models:

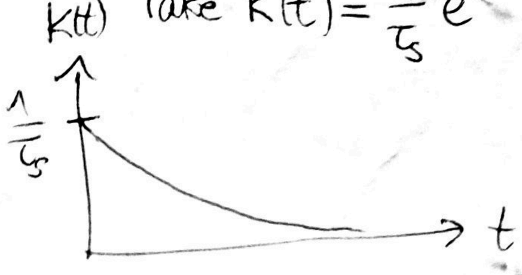
+ analytic computations are possible
+ seen as the average over trials, therefore takes into account stochasticity due to ignorance about all inputs and connections
- looses information encoded in time correlations between spikes
+ Electrophysiology/fMRI rather measures response of a group of neurons than of a single neuron



$$I(t) = \sum_b w_b \sum_{\substack{\text{spikes at} \\ t_i<t}} \underbrace{K(t-t_i)}_{\substack{\text{Current at soma due to} \\ \text{incoming spike at time } t_i}} = \sum_b \int_{-\infty}^{t} ds\, K(t-s) \underbrace{\sum_{\substack{t_i \text{ spiketimes} \\ \text{at input } b}} \delta(s-t_i)}_{\substack{u_b(s) \\ \text{firing rate at input } b}}$$

input weights
{ $w<0$: inhibitory
{ $w>0$: excitatory

Take $K(t) = \frac{1}{\tau_s} e^{-t/\tau_s}$

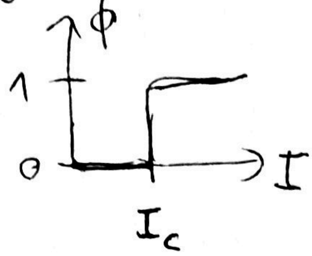


$$\tau_s \partial_t I(t) = \sum_b w_b \tau_s \underbrace{K(0)}_{\frac{1}{\tau_s}} u_b(t) + \tau_s \sum_b w_b \int_{-\infty}^{t} ds \, \underbrace{K'(t-s)}_{-\frac{1}{\tau_s} K(t-s)} u_b(s)$$

$$= \sum_b w_b u_b(t) \cancel{+} - I(t)$$

Assumption: firing rate due to current $I(t)$ is given by $u = \phi(I(t))$

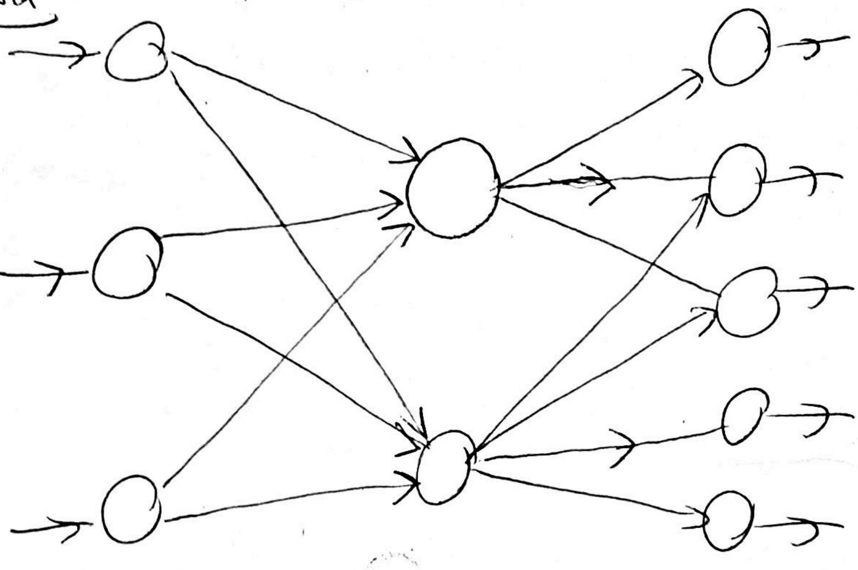
with a non linear function



Often $\phi(I) = \tanh(I)$ $(\Leftrightarrow I_c = 0)$

feed ~~f~~forward

In



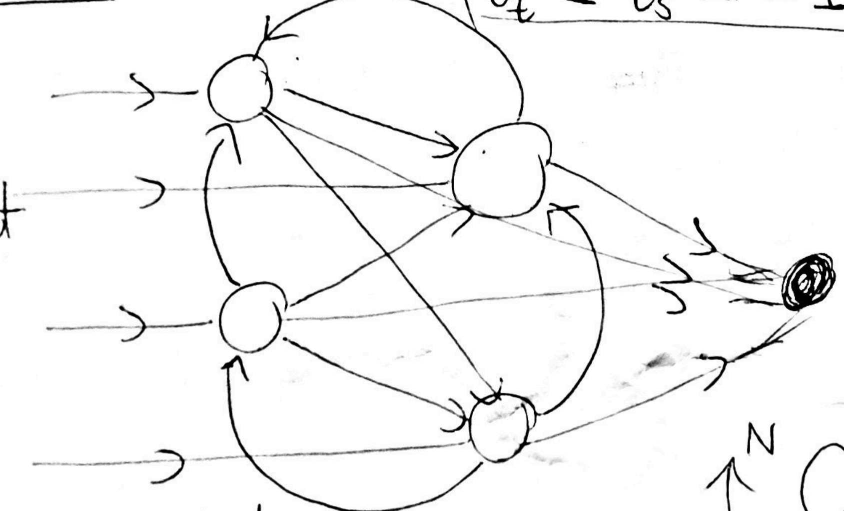
out

recurrent

$$\boxed{\partial_t \vec{I} \tau_s = -\vec{I} + W \phi(I)} + \vec{I}_{input}$$

connectivity matrix

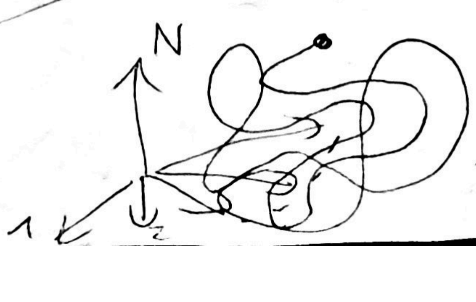
Input



Output reading

$z_t = \vec{w}_{out} \cdot \vec{I}_t$

Neural State Space:



Usually a low dim space