

Graduate Texts in Mathematics

V.I. Arnold

Mathematical Methods of Classical Mechanics

Second Edition

 Springer

PROBLEM.²³ A desert animal has to cover great distances between sources of water. How does the maximal time the animal can run depend on the size L of the animal?

PROBLEM.²⁴ How does the running velocity of an animal on level ground and uphill depend on the size L of the animal?

PROBLEM.^{24a} How does the height of an animal's jump depend on its size?

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ANSWER. It is directly proportional to L .

Solution. The store of water is proportional to the volume of the body, i.e., L^3 ; the evaporation is proportional to the surface area, i.e., L^2 . Therefore, the maximal time of a run from one source to another is directly proportional to L .

We notice that the maximal distance an animal can run also grows proportionally to L (cf. the following problem).

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PROBLEM.²⁴ How does the running velocity of an animal on level ground and uphill depend on the size L of the animal?

Solution. The power developed by the animal is proportional to L^2 (the percentage used by muscle is constant at about 25%, the other 75% of the chemical energy is converted to heat; the heat output is proportional to the body surface, i.e., L^2 , which means that the effective power is proportional to L^2).

The force of air resistance is directly proportional to the square of the velocity and the area of a cross-section; the power spent on overcoming it is therefore proportional to $v^2 L^2 v$. Therefore, $v^3 L^2 \sim L^2$, so $v \sim L^0$. In fact, the running velocity on level ground, no smaller for a rabbit than for a horse, in practice does not specifically depend on the size.

The power necessary to run uphill is $mgv \sim L^3 v$; since the generated power is $\sim L^2$, we find that $v \sim L^{-1}$. In fact, a dog easily runs up a hill, while a horse slows its pace.

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PROBLEM.^{24a} How does the height of an animal's jump depend on its size?

ANSWER. $\sim L^0$.

Solution. For a jump of height h one needs energy proportional to $L^3 h$, and the work accomplished by muscular strength F is proportional to FL . The force F is proportional to L^2 (since the strength of bones is proportional to their section). Therefore, $L^3 h \sim L^2 L$, i.e., the height of a jump does not depend on the size of the animal. In fact, a jerboa and a kangaroo can jump to approximately the same height.



Wet mammals shake at tuned frequencies to dry (2012)

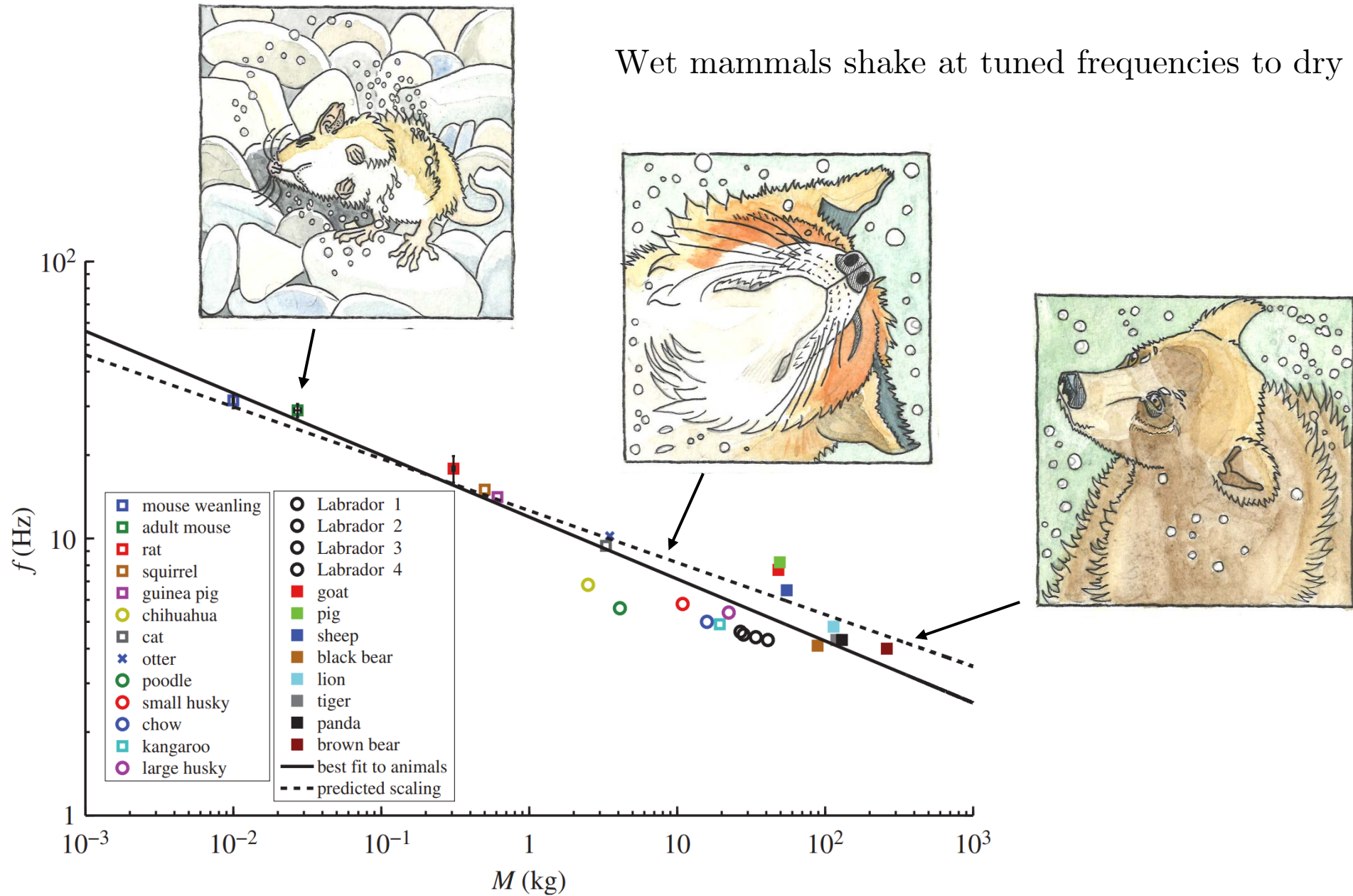
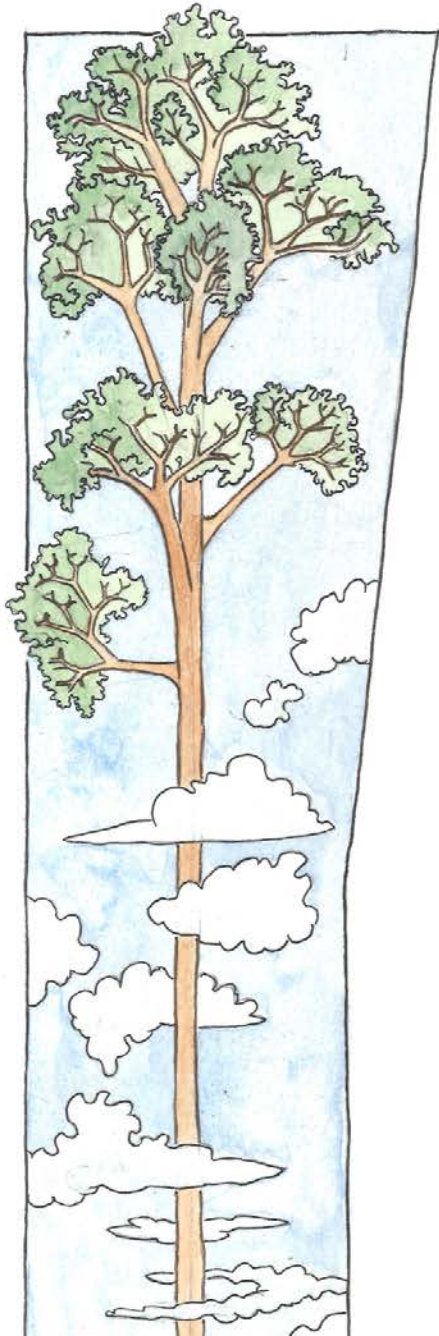
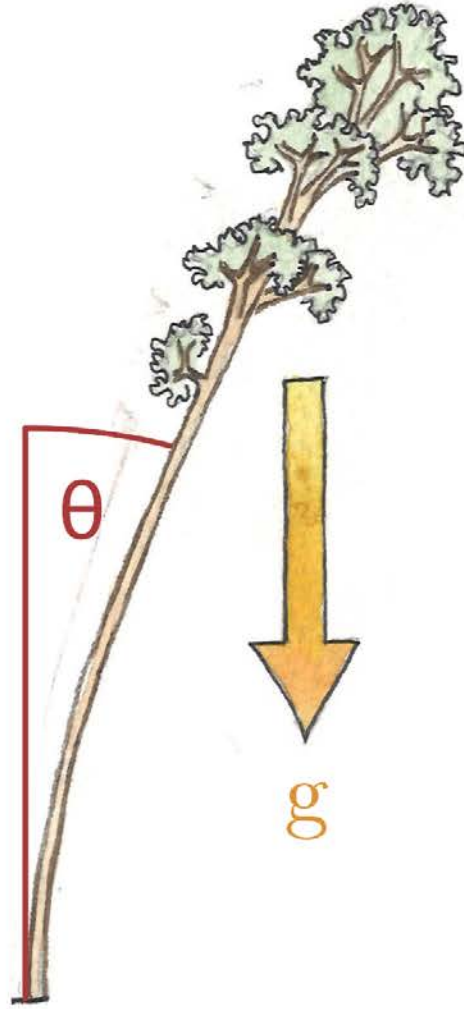


Figure 3. The relation between shaking frequency f and animal radius R . Dogs are denoted by a circle, other mammals by a square and the semi-aquatic otter by an X. Best fit is given in equation (3.1). Error bars indicate the standard deviation of measurement.

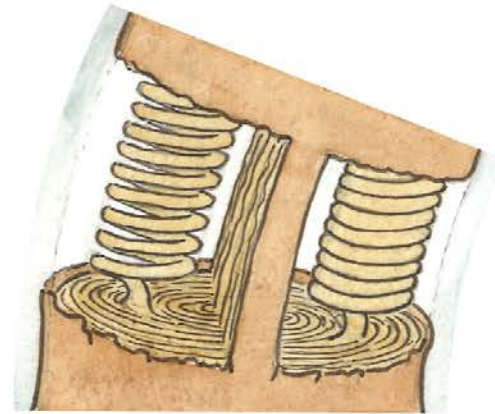
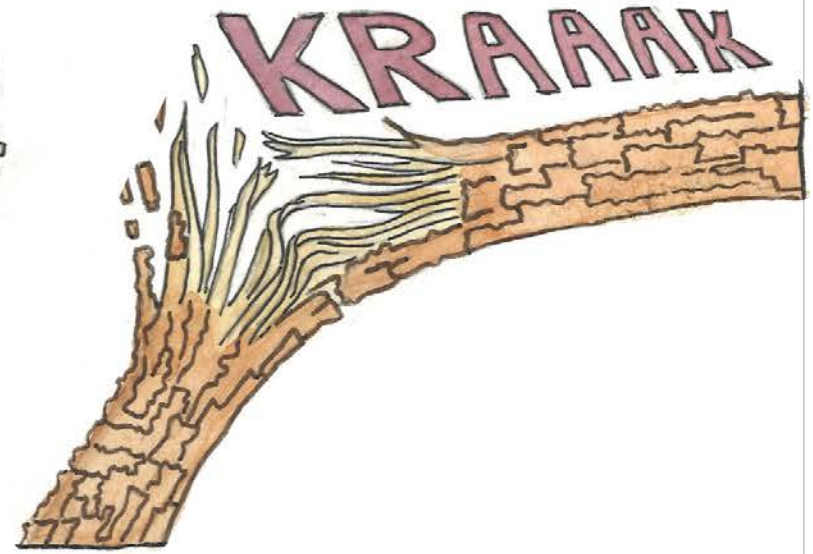


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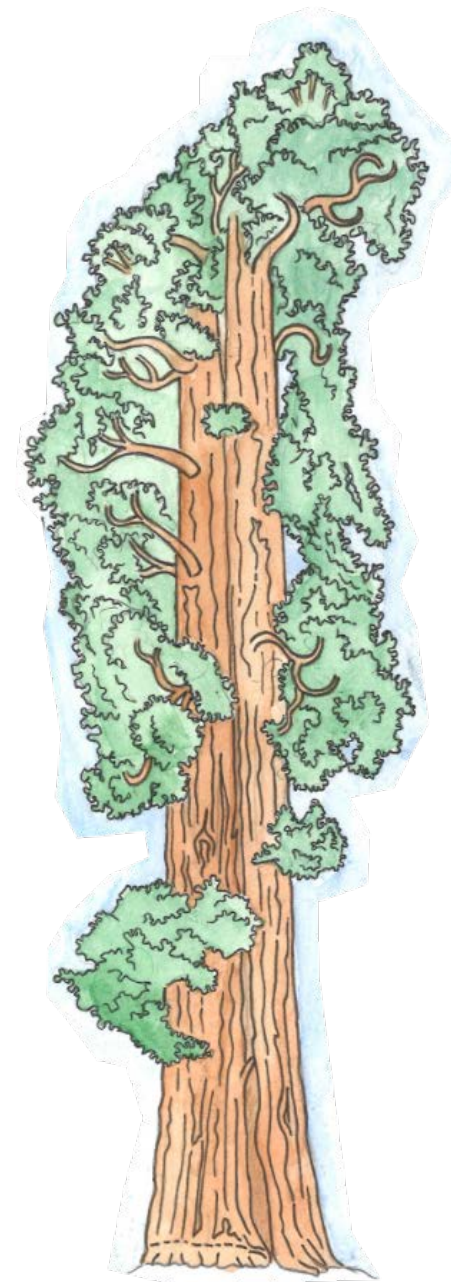
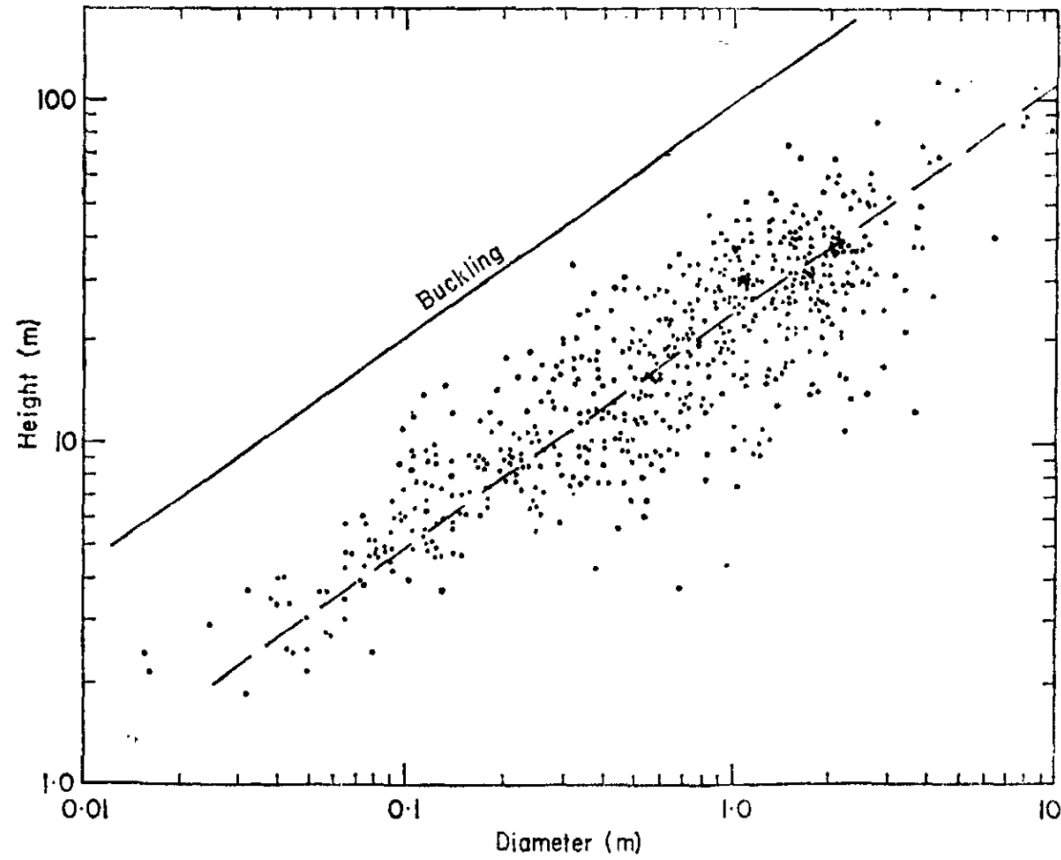
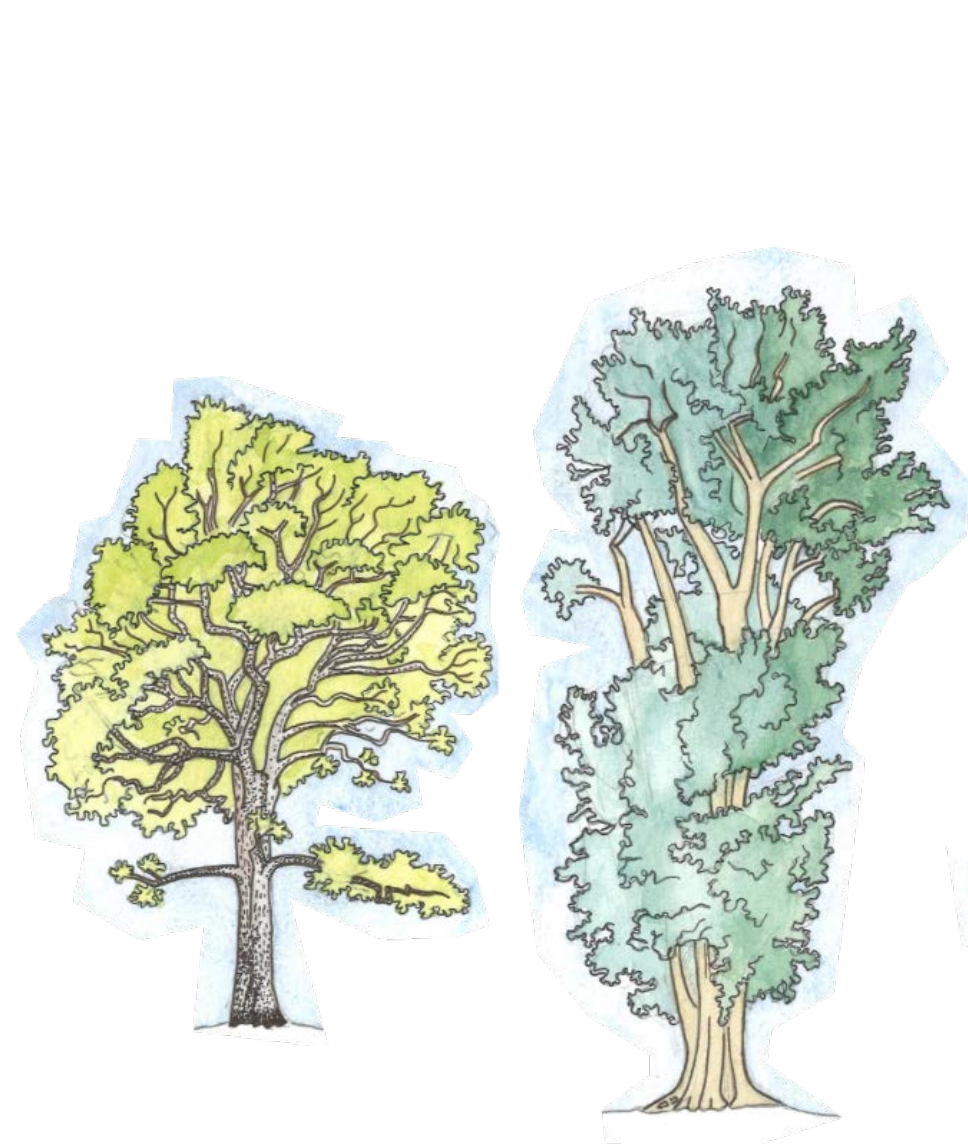
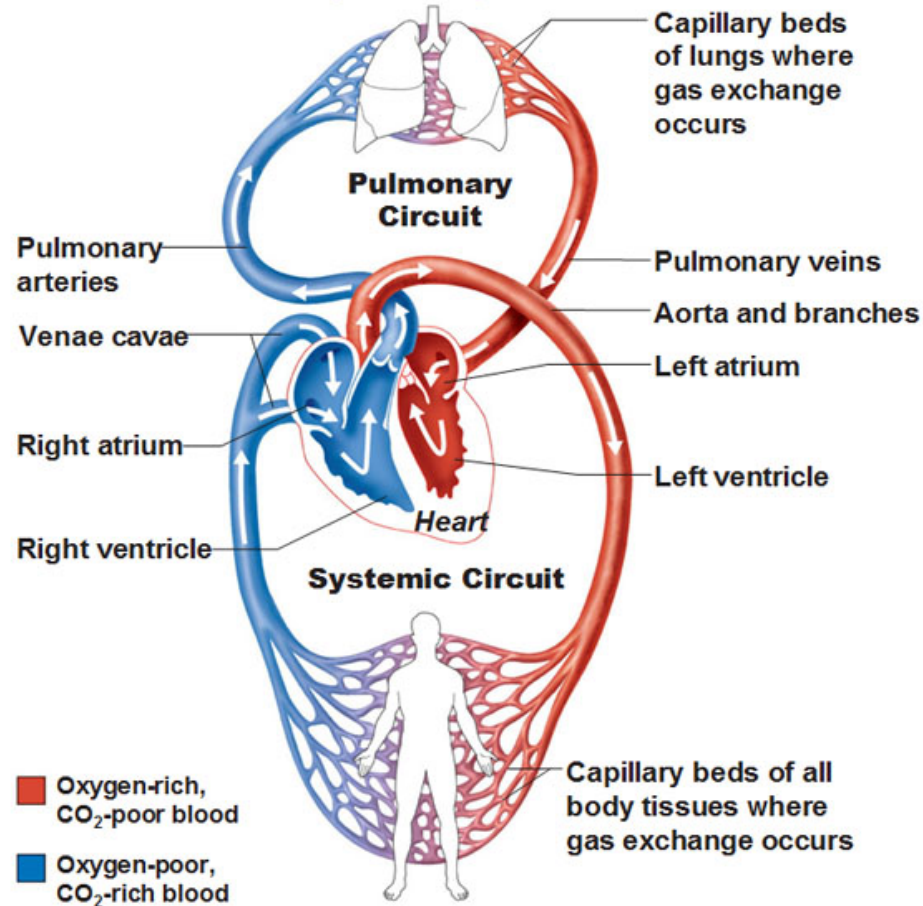


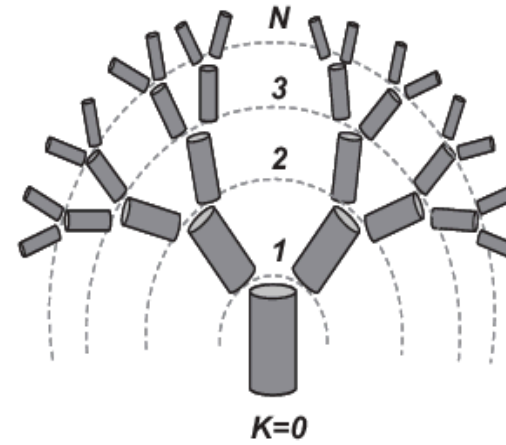
FIG. 5. Overall height vs. base diameter for 576 record trees representing nearly every species found in the United States. Data from the American Forestry Association's Social Register of Big Trees. The argument is that trunk proportions are limited by elastic buckling criteria, since no points lie to the left of the solid line (from McMahon, 1973).

The Heart as a Double Pump The Pulmonary and Systemic Circuits

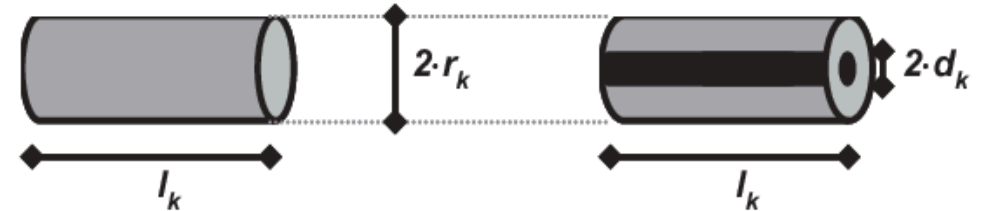
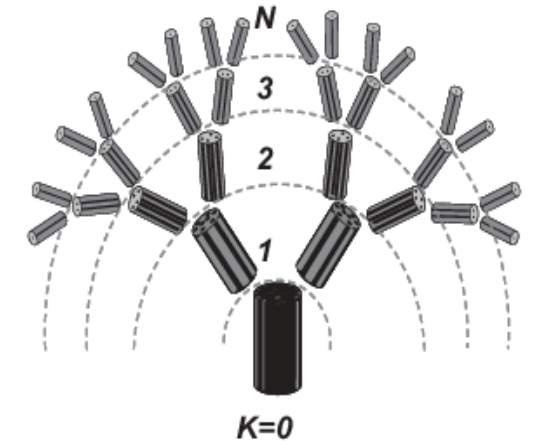


<https://m.newhealthadvisor.org/blood-flow-through-the-heart.html>

WBE 97 (cardio-vascular system)



WBE 99 (xylem vascular system)



$$\gamma = \frac{l_{k+1}}{l_k} = n^{-1/3} \quad \text{Eq. 1}$$

$$\beta = \frac{r_{k+1}}{r_k} = n^{-a/2} \quad \text{Eq. 2}$$

$$\bar{\beta} = \frac{d_{k+1}}{d_k} = n^{-\bar{a}/2} \quad \text{Eq. 3}$$

l_k = branch and conduit length
 r_k = conduit radius (WBE 97) and cibranch radius (WBE 99)
 d_k = conduit radius (WBE 99)
 n = branching ratio (furcation)
 k = branching generations

La loi de Kleiber

La controverse du modèle WBE

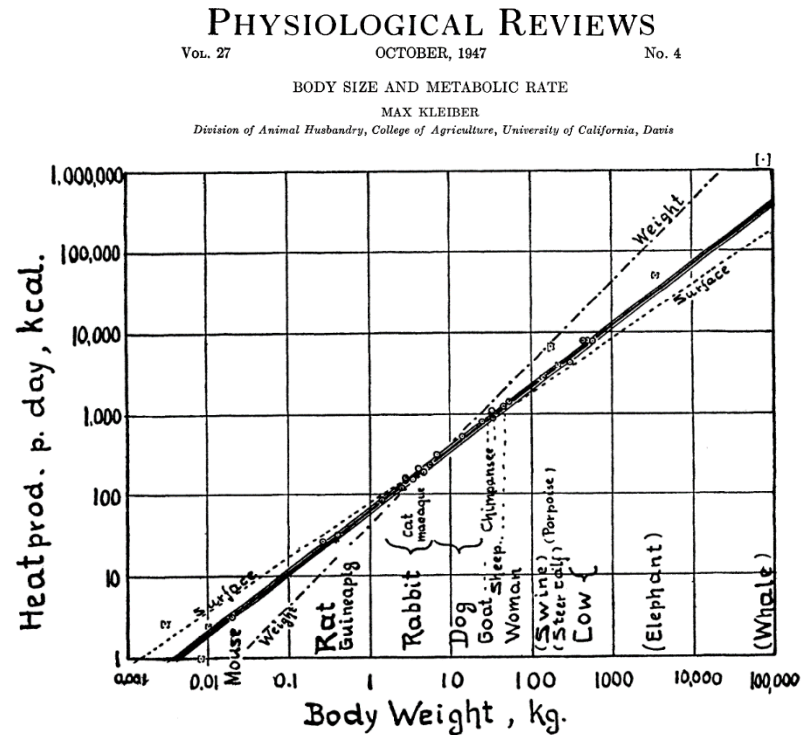


Fig. 1. Log. metabol. rate/log body weight

A General Model for the Origin of Allometric Scaling Laws in Biology

Geoffrey B. West, James H. Brown,* Brian J. Enquist

Allometric scaling relations, including the 3/4 power law for metabolic rates, are characteristic of all organisms and are here derived from a general model that describes how essential materials are transported through space-filling fractal networks of branching tubes. The model assumes that the energy dissipated is minimized and that the terminal tubes do not vary with body size. It provides a complete analysis of scaling relations for mammalian circulatory systems that are in agreement with data. More generally, the model predicts structural and functional properties of vertebrate cardiovascular and respiratory systems, plant vascular systems, insect tracheal tubes, and other distribution networks.

FORUM

Is West, Brown and Enquist's model of allometric scaling mathematically correct and biologically relevant?

J. KOZŁOWSKI*† and M. KONARZEWSKI‡

*Institute of Environmental Sciences, Jagiellonian University, Gronostajowa 7, 30-387 Krakow, Poland, and †Institute of Biology, University of Białystok, Świerkowa 20B, 15-950 Białystok, Poland

FORUM

Yes, West, Brown and Enquist's model of allometric scaling is both mathematically correct and biologically relevant

J. H. BROWN,*†‡ GEOFFREY B. WEST†‡ and B. J. ENQUIST§

*Department of Biology, University of New Mexico, Albuquerque, NM 87131, USA, †Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA, ‡Theoretical Division, MS B285, Los Alamos National Laboratory, Los Alamos, NM 87545, USA, §Department of Ecology and Evolutionary Biology, University of Arizona, Tucson, AZ 85721, USA

Mammalian basal metabolic rate is proportional to body mass^{2/3}

Craig R. White* and Roger S. Seymour

Department of Environmental Biology, University of Adelaide, Adelaide 5005, Australia

Edited by Knut Schmidt-Nielsen, Duke University, Durham, NC, and approved December 27, 2002 (received for review October 23, 2002)

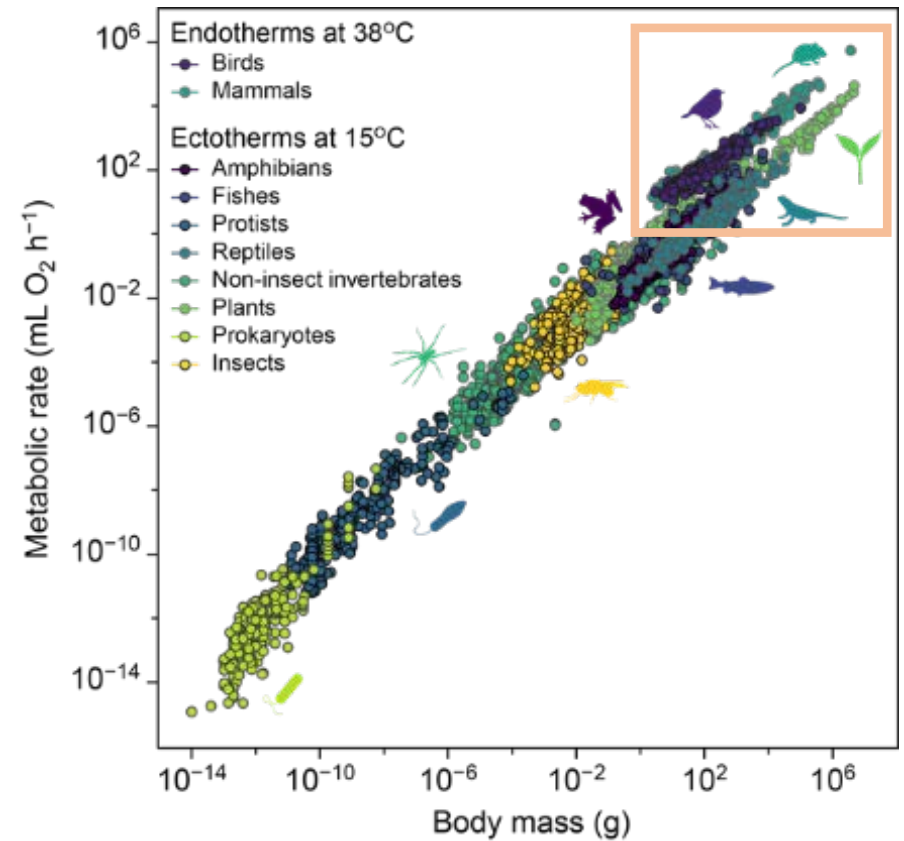
Vol 46:1 April 2010 | doi:10.1038/nature08920

nature

LETTERS

Curvature in metabolic scaling

Tom Kolokotronis¹, Van Savage², Eric J. Deeds¹ & Walter Fontana¹



Norin (2022)