Nonequilibrium signatures and phase transitions in active matter and beyond

PhD thesis of D. Martin supervised by J. Tailleur

Special thanks: C. Nardini

Collaborators: T. Arnoulx de Pirey, D. Bartolo, H. Chaté, D. Geyer, Y. Kafri, M. Kardar, C. Nardini, J. O'byrne, A. Solon, F. Van Wijland

Laboratoire MSC
université de Paris

October 8, 2021

Microscopic Active Matter

- Particles exerting self-propulsion forces on their medium

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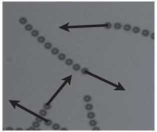
Synthetic

- ⁎ Physical mechanism

Quincke rollers



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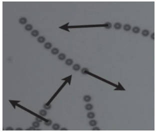

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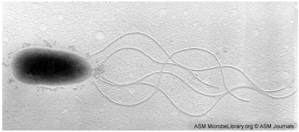
Living entities

✱ Biological mechanism

Bacterium



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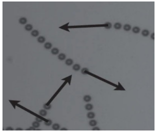

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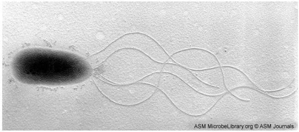

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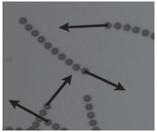

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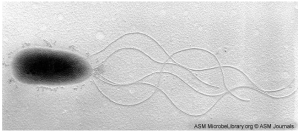

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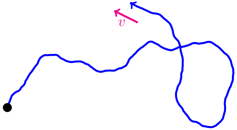

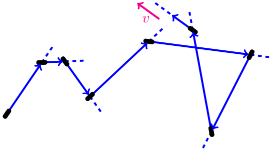

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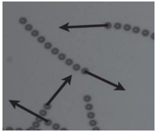

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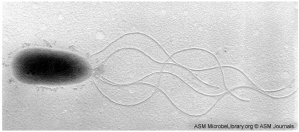

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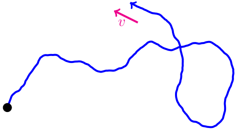

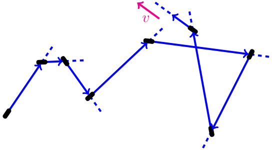

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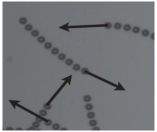

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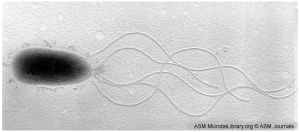

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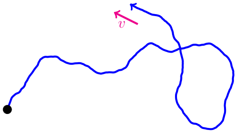

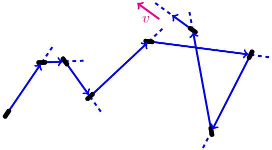

RTP

- unusual fluctuations ⟶ algebraic computations challenging
- steady-state distribution: unknown
- departure from equilibrium: unquantified

Macroscopic Active Matter

- Active systems $\longrightarrow$ ubiquitous in nature

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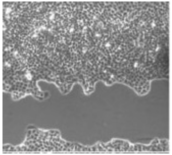
[Ballerini et al, *PNAS* 105, 2008]



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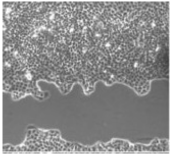

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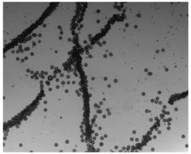

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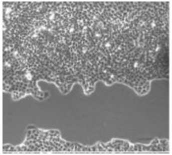
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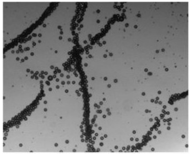
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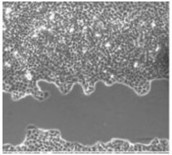

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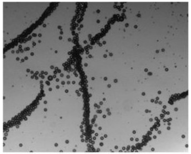

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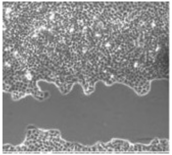

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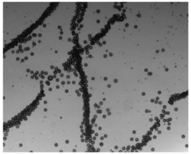

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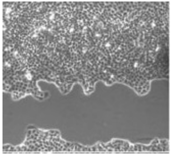

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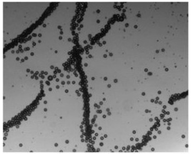

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Motility-Induced Phase Separation (MIPS)
repulsive forces

Flocking transition
alignment

Statistical physics of Active Matter: MIPS

- Phase separation at equilibrium: attractive forces vs thermal noise
  $\hookrightarrow$ Low T $\longrightarrow$ cohesion wins: liquid-gas coexistence

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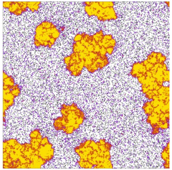

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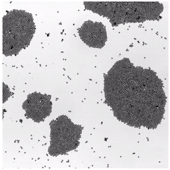

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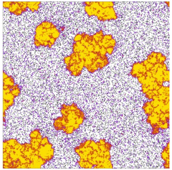

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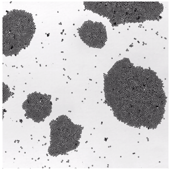

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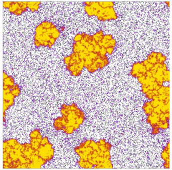

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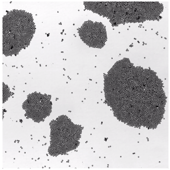

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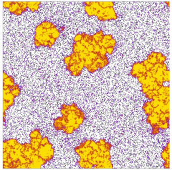

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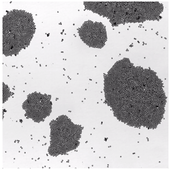

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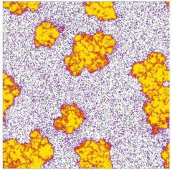

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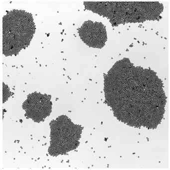

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- MIPS in self-propelled spheres: starts to be understood

- MIPS for generic interactions: more complex

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- Modelling flocking transition $\longrightarrow$ aligning interactions

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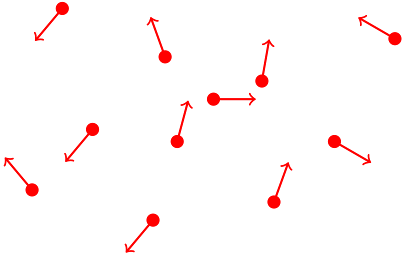

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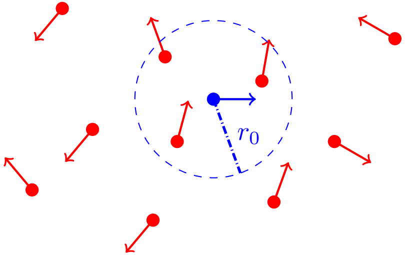

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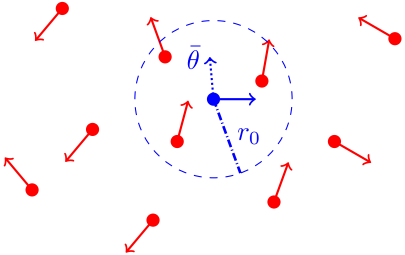

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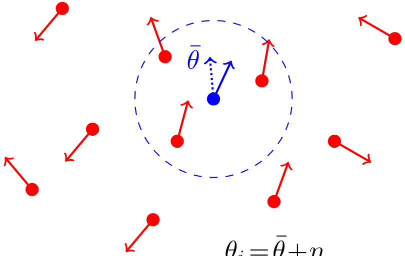

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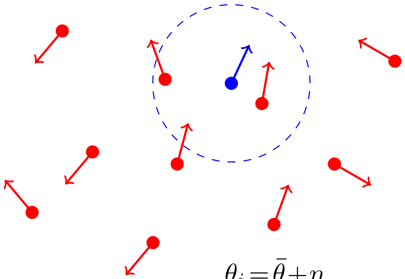

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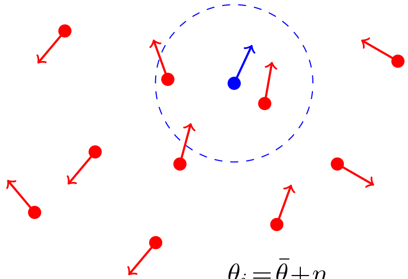

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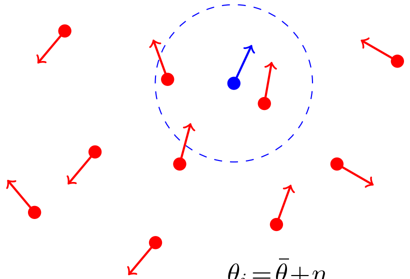

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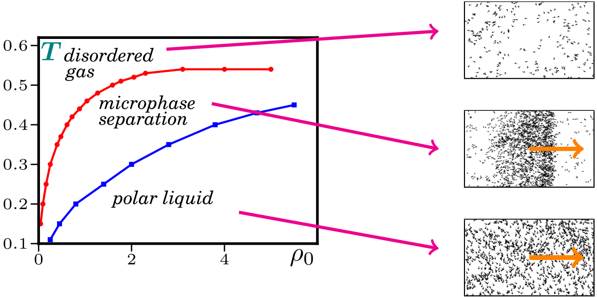
- Phase diagram [Solon et al, PRL 114, 2015]



Statistical physics of Active Matter: flocking

- Vicsek Model ⟶ relevant for experiments

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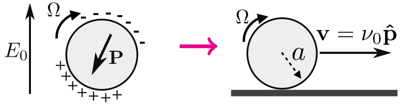

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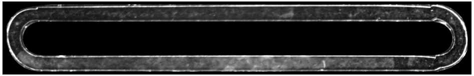

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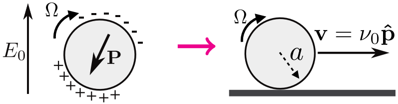

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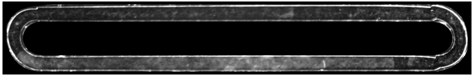

- hydrodynamic + electrostatic interactions → effective alignment
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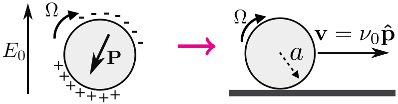
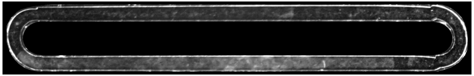


[Bricard et al, *Nature* 503]

⇒ Pack them



- hydrodynamic + electrostatic interactions → effective alignment

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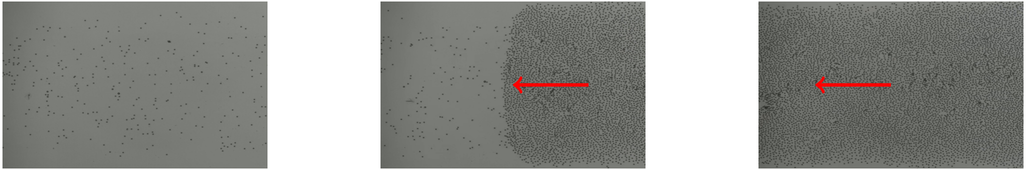

Disordered gas | Polar bands | Ordered flock

Statistical physics of Active Matter: flocking

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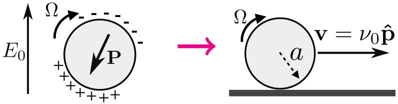

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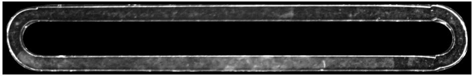

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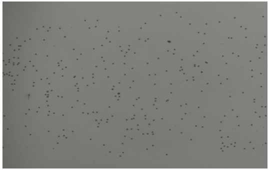

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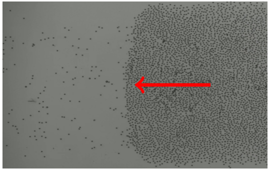
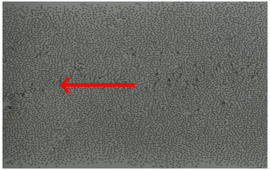


Polar bands



Ordered flock

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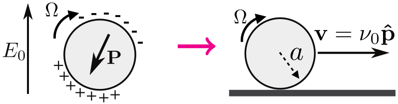

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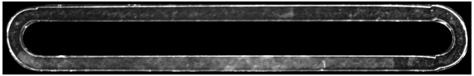

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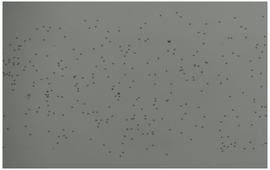

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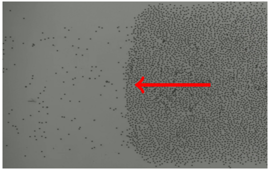

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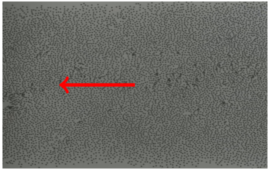

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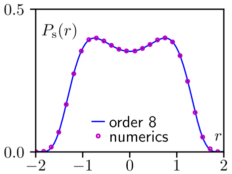
What lies beyond for more complex systems ?

The four axes of this thesis

Exact results for a single active particle



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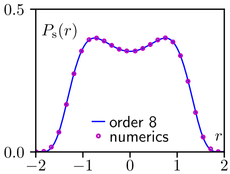
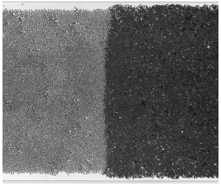


MIPS in dense polar flocks



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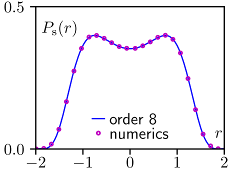
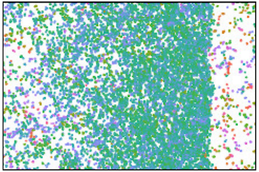


Fluctuation-induced first-order flocking



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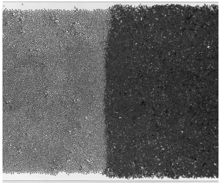

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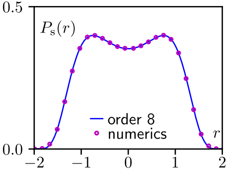

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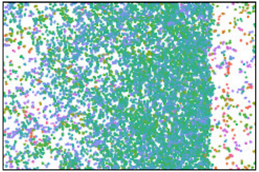

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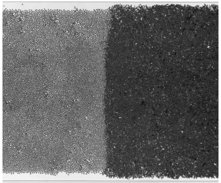
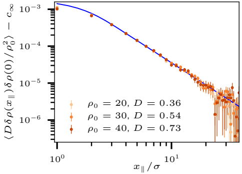


Anisotropy-induced long-ranged correlations



Part I

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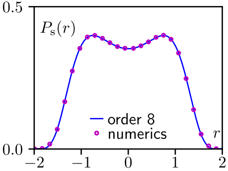

Part III

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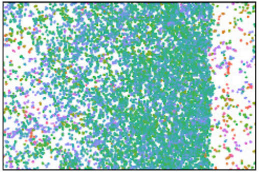

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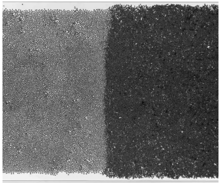

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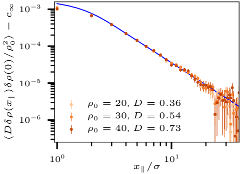

Microscopic Active Matter: exact approaches

- Simplest model ⟶ Active Ornstein-Ulhenbeck Particles (AOUPs) [Fodor et al, *PRL* 117, 2016]

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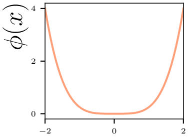

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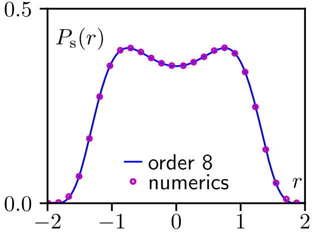
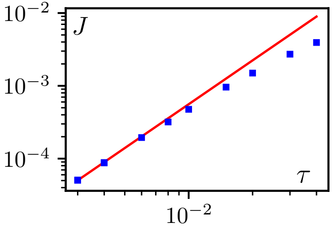
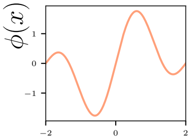


ratchet current $J \propto \tau^2$





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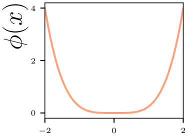

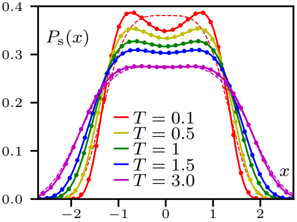

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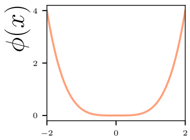

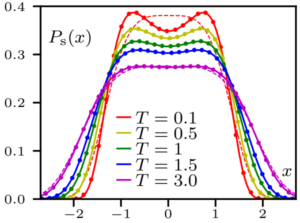

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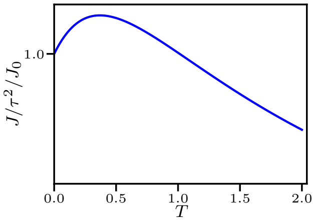

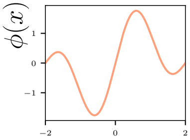

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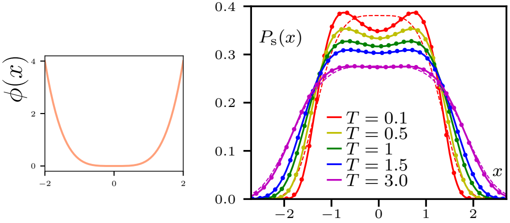

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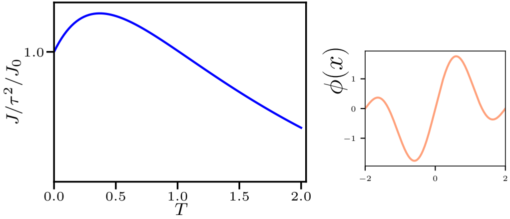
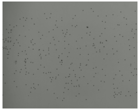
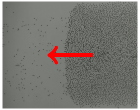


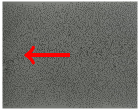
- Non trivial interplay between active and passive noises

Part II: Motility-induced solidification in roller flocks

Low density: Vicsek







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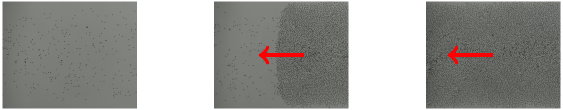
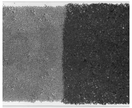


High density: beyond Vicsek



New phase transition
active solidification

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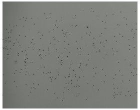

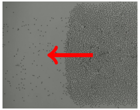

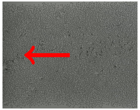

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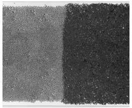

New phase transition
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- What is happening ?

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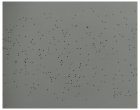

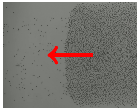

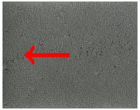

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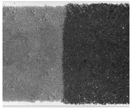

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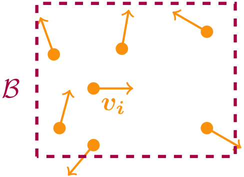

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Part II: Motility-induced solidification in roller flocks

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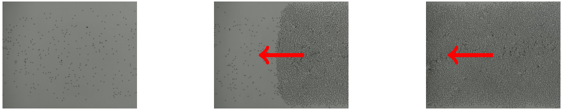
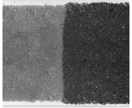


High density: beyond Vicsek



New phase transition
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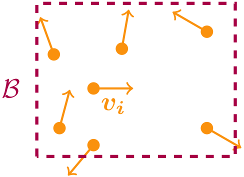

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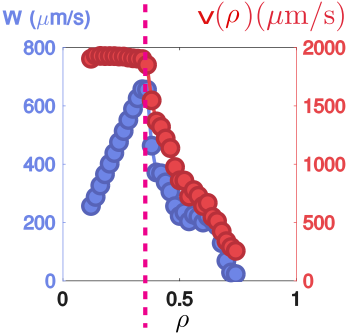
Vicsek physics | Beyond Vicsek



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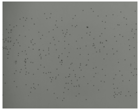

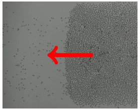

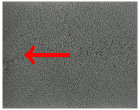

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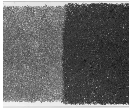

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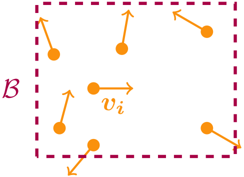

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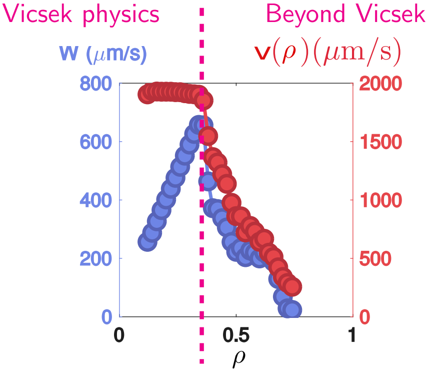

⟹ Is it MIPS at play in a flock ?

Phenomenological hydrodynamics

- Standard hydrodynamic of the Vicsek model for $\rho = \langle \sum_i \delta(r - r_i) \rangle$ and $W = \langle \sum_i v_i \delta(r - r_i) \rangle$

$$\partial_t \rho = D_\rho \partial_{xx} \rho - \partial_x W \tag{1}$$

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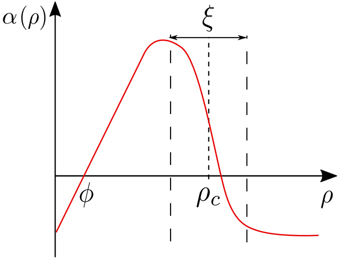
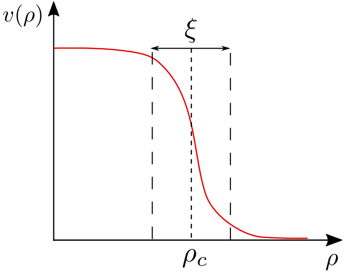


Ordering drop



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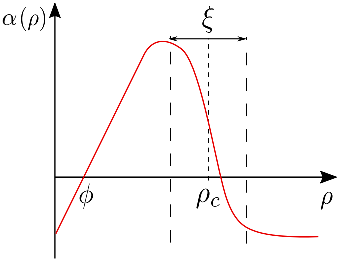

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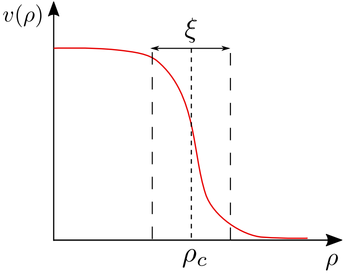
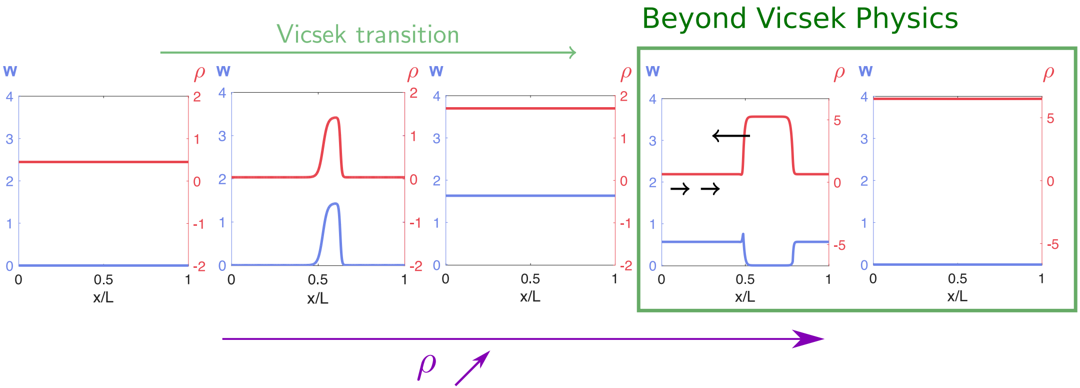


Velocity drop

Postulated phenomenologically, could be rigorously derived

- New phase transition at high density



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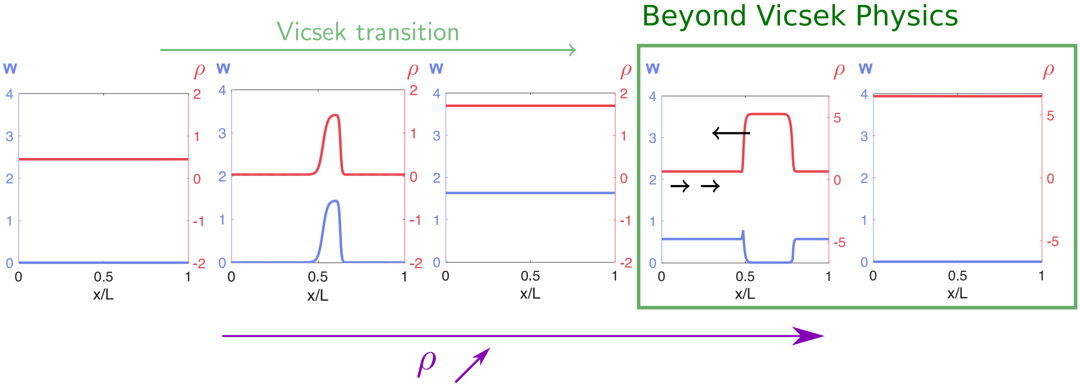

- MIPS-like transition: linear instability, lever rule, hysteresis loops, coarsening dynamics.

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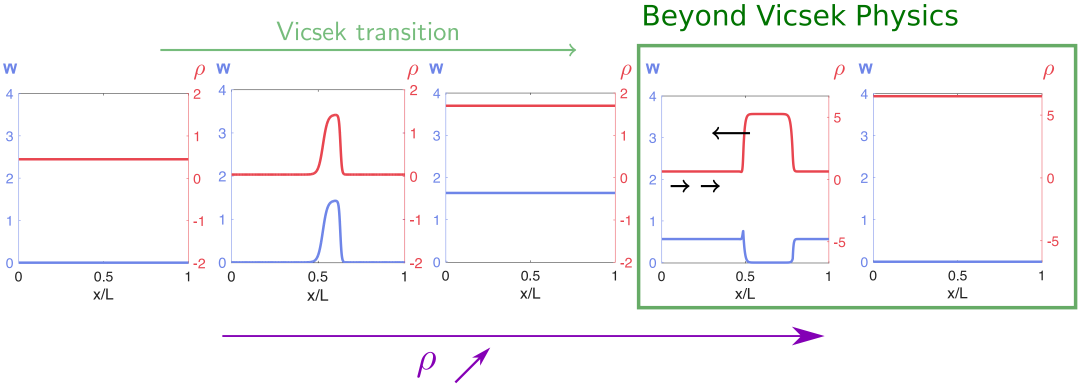

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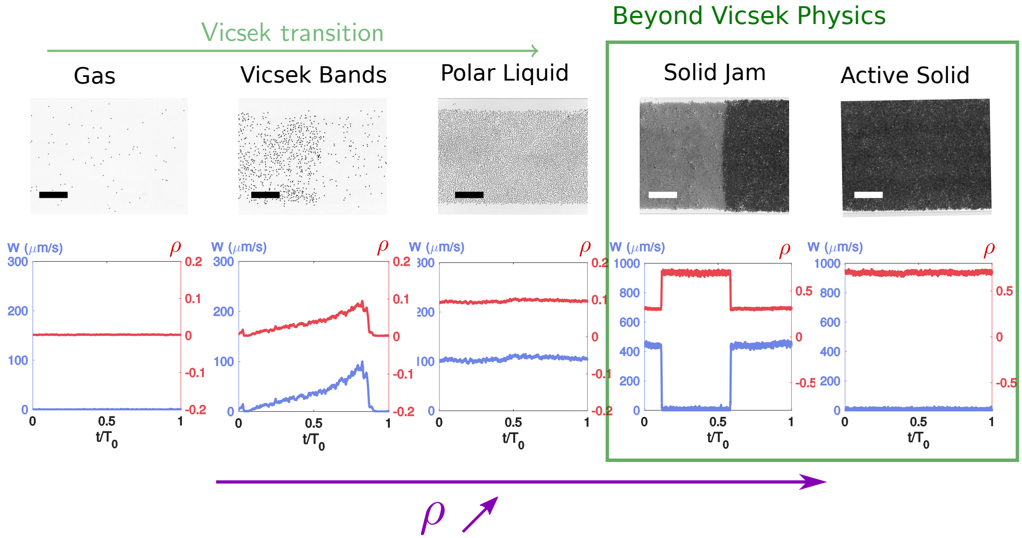
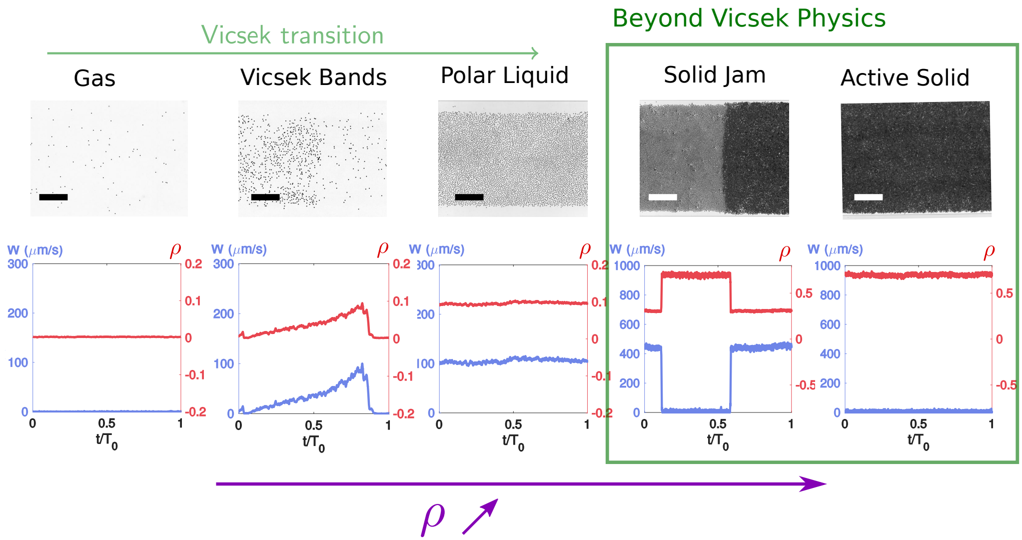


Vicsek transition
Beyond Vicsek Physics
Gas
Vicsek Bands
Polar Liquid
Solid Jam
Active Solid
W (μm/s)
ρ
t/T0
ρ ↗

Quincke rollers experiments



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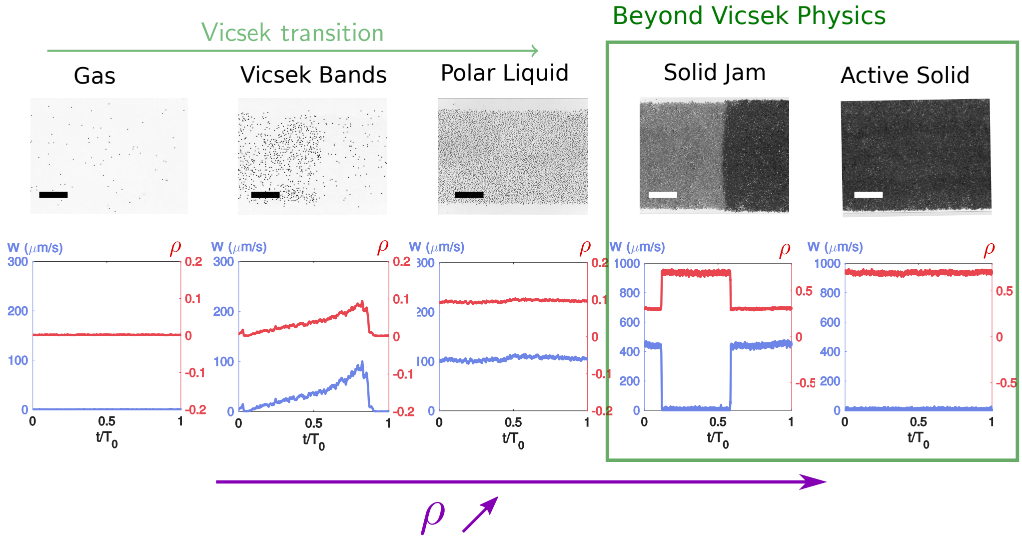

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- Described by MIPS occurring in a polar liquid
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- Agreement of phenomenological hydrodynamics and experiments beyond the phase diagram
  - $\hookrightarrow$ Lever rule
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- Emergence of collective motion: 1^{st} or 2^{nd} order ?

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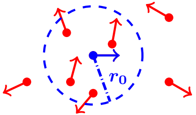
First order / coexistence

- Vicsek: metric alignment



[Grégoire et al, PRL 92, 2004]

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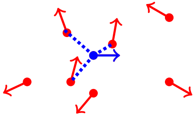

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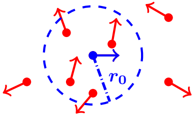

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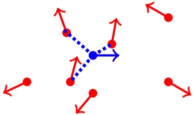

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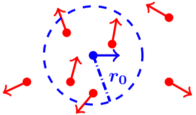

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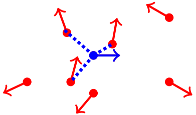

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⟶ However, microscopic simulations disagree: why? Is it generic?

The Active Ising Model [Solon et al, PRL 111, 2013]

- specific system but generic results
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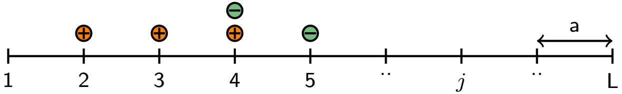

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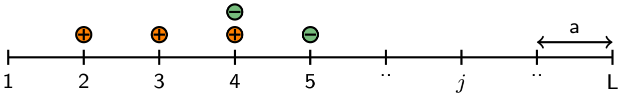

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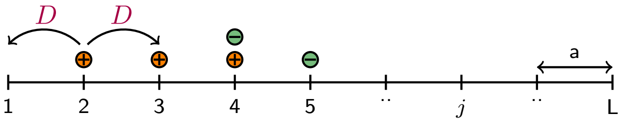

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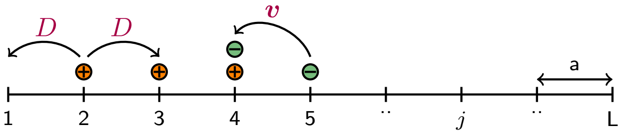

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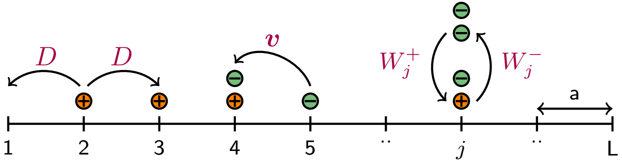

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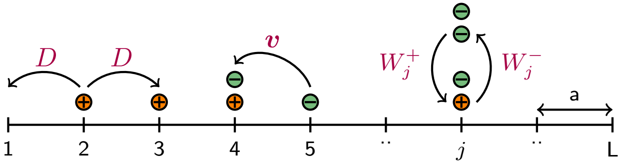

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Master equation + Mean-field approximation

⇓

Hydrodynamics for $\boldsymbol{\rho}$ and $\boldsymbol{m}$

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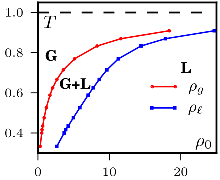

⇒ At odds with microscopic simulations

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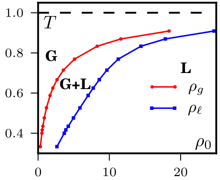

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- What did mean-field miss ?

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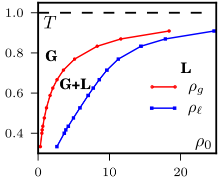

⟹ At odds with microscopic simulations

- What did mean-field miss ? Fluctuations

The Active Ising Model [Solon et al, PRL 111, 2013]

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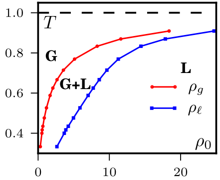

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- What did mean-field miss ? Fluctuations
  ↪ renormalize the Landau terms in the dynamics of $\langle\rho\rangle$ and $\langle m\rangle$ $\mathcal{F} \longrightarrow \mathcal{F}_{MF} + \Delta\mathcal{F}$

Quasi-linear renormalization

- Fluctuations $\rho = \rho_0 + \delta\rho$ and $m = m_0 + \delta m$ around homogeneous mean field

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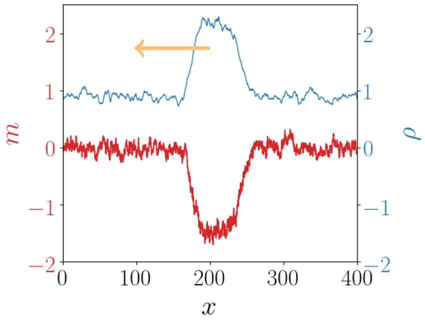

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MF hydrodynamics with deceptive continuous transition

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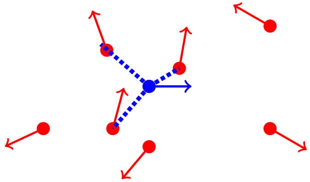

What about 'topological' or 'metric-free' models ?

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PNAS

Interaction ruling animal collective behavior depends on topological rather than metric distance: Evidence from a field study

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Voronoi neighbours

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⇩

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Protected against fluctuations ?

The fluctuating topological hydrodynamics

- Renormalization of linear Landau term

$$\alpha \longrightarrow \alpha + \frac{\sigma\Gamma}{k}\, g\left(\beta, \frac{\Gamma k}{v\boldsymbol{\rho}}, \frac{\Gamma D}{v^2}\right) \Longrightarrow \boldsymbol{\alpha} \text{ density dependent}$$

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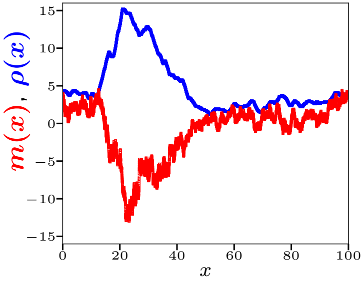

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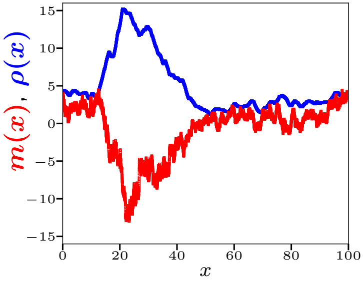

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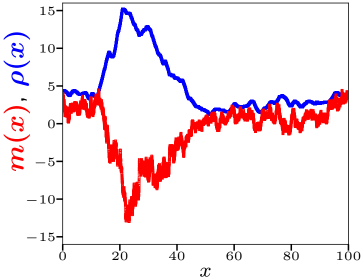

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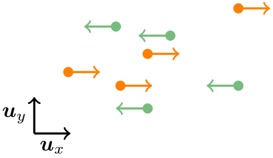
  $\hookrightarrow$ Let's try to see if it is robust for microscopic models !

The microscopic topological dynamics

- Microscopic dynamics of the topological Active Ising Model
  - Off-lattice Langevin particles
  - Each particles carries a spin

$$\dot{\boldsymbol{r}}_j = s_j v \, \boldsymbol{u}_x + \sqrt{2D} \, \boldsymbol{\eta}_j$$



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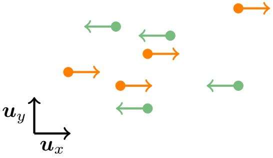
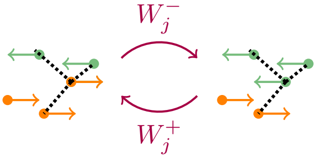
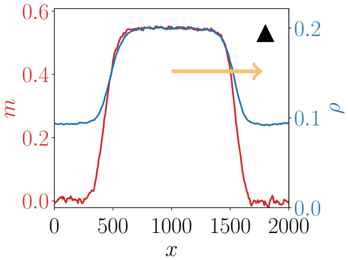
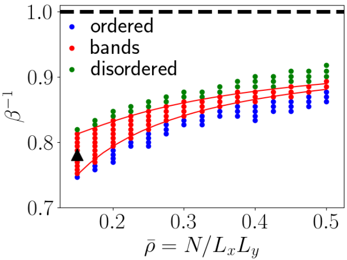




  - Flipping rates $W_j^{\pm} = \Gamma \exp(\pm \beta \tilde{m}_j)$ with $\tilde{m}_j =$ averaged magnetization of k-nearest neighbours

- Results of the microscopic topological Active Ising Model





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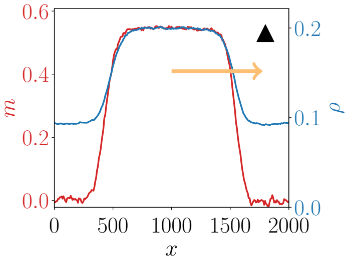

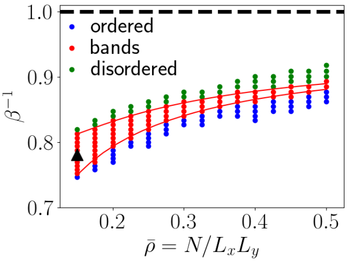

↳ Same fate for microscopic topological models

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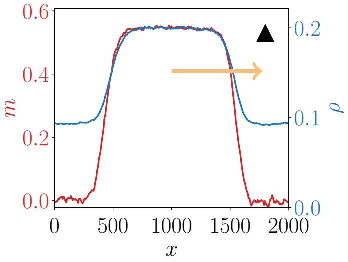

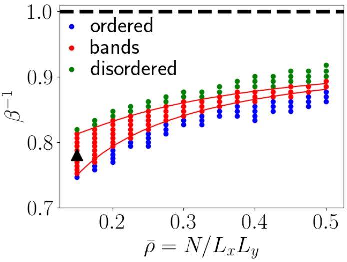

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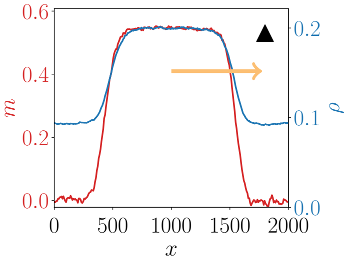

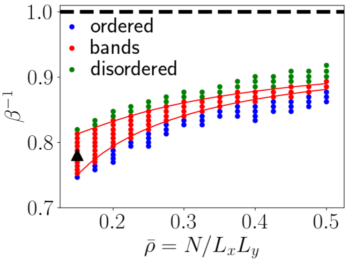

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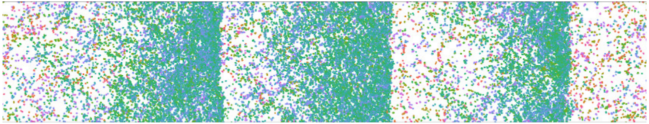
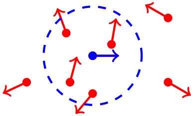
↳ Holds also for topological Vicsek Model



Revisiting the classification

First order / coexistence

- Vicsek: metrical alignment



Second order / continuous

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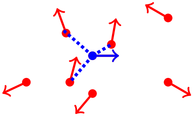

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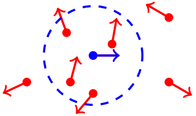

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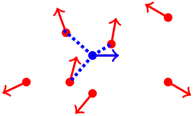

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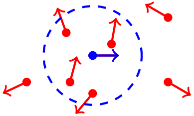

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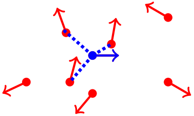

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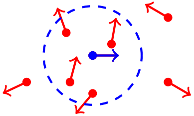

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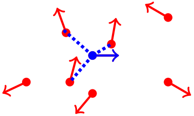

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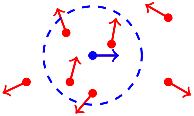

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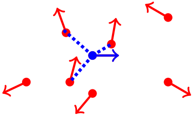

- ~~Active Ising Model: hydrodynamic~~
- Fully connected models

- Are all models of collective motion first order ?

  ↪ No, fully connected alignment ⇔ continuous transition

$$\alpha \longrightarrow \alpha + \frac{\sigma\Gamma}{N}\, g\left(\beta, \frac{\Gamma D}{v^2}\right) \Longrightarrow$$

- ✱ no dependence on local density
- ✱ vanishes as the number of particles diverges

Part III: conclusion

- Fluctuations renormalize T_c making it density dependent

- $T_c(\rho)$ turns a deceptive continuous transition into a first order scenario

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- Quantification of departure from equilibrium in AOUP
  - ↪ overdamped active particle: go to underdamped scenario
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- List of publications

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D. Martin, T. Arnoulx de Pirey, JSTAT Mech. **4**, 043205 (2021)
D. Geyer, D. Martin, J. Tailleur, D. Bartolo, Phys. Rev. X **9**, 031043, (2019)
D. Martin, H. Chaté, C. Nardini, A. Solon, J. Tailleur and F. Van Wijland, Phys. Rev. Lett. 126 148001 (2021)